Transfer Pricing and Location of Intangibles – Spillover and Tax Avoidance through Profit Shifting

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Abstract

This study examines how spillovers affect a multinational company’s choice of an intangible’s location and the corresponding royalty-based transfer price for using this intangible. Our model examines a company with a domestic division in a high-tax country and a foreign division in a low-tax country. In contrast to previous studies, our analysis incorporates spillovers that each division generates on the other division’s profit. When determining the intangible’s location and the royalty rate, the company trades off tax minimization and spillover internalization. By locating the intangible abroad, the company reduces its tax liability. When the intangible is located in the domestic division, spillover internalization is more efficient. Given a high domestic division’s spillover, the company locates the intangible in the domestic division. A ‘home bias’ occurs. We show how variations in tax regulations affect the intangible’s location. A narrower arm’s length range or higher tax rate differentials reduce the ‘home bias’.

Keywords. profit shifting, intangibles, spillover, transfer pricing, location choice
JEL Classifications. F23, L24, H26, O34

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1 Introduction

The location of an intangible and the corresponding transfer pricing affect a multinational company’s (MNC’s) tax liability. MNCs frequently use transfer pricing to shift profits to low-tax countries (e.g., De Simone et al., 2017; Blouin et al., 2018) and thereby reduce their tax liabilities. By locating intangibles in low-tax countries and subsequently stipulating transfer prices, MNCs can legally reduce tax payments in high-tax countries to achieve low effective tax rates. MNCs’ low effective tax rates have drawn public attention, and as a result, the OECD recently implemented the outcomes of the BEPS project in the transfer-pricing guidelines to curtail MNCs’ profit-shifting possibilities. In contrast to the public media attention, which indicates that MNCs locate intangibles offshore to reduce tax liability (The Guardian, 2009), empirical evidence reveals a large ‘home bias’ for intangibles. Karkinsky and Riedel (2012) show that the average European MNC files 57.1 percent of its annual patent applications from the parent country. For trademarks, Heckemeyer et al. (2018) find an even stronger ‘home bias’. They document that 95.3 percent of the U.S. trademarks registered at the United States Patent and Trademark Office between 2003 and 2012 are owned by U.S. entities of the S&P 500.

This empirically documented ‘home bias’ suggests that MNCs also consider other factors such as an intangible’s characteristics when they determine the locations of their intangibles. The unique nature and especially the public good character of intangibles allow MNCs considerable discretion in the location of intangibles and the corresponding transfer pricing. MNCs can easily relocate an intangible without physically moving assets, the use of existing intangibles entails no or negligible marginal costs, and an intangible is usually nonexclusive in its consumption (Lev, 2001). Additionally, spillovers or network effects typically occur (Lev, 2001), so the uses of an intangible by several divisions are not independent from one another. These interdependencies may appear as spillovers from one division’s activities that involve the intangible to other divisions’ profits. The

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1 The Guardian (2009) reports that AstraZeneca, Diageo, GlaxoSmithKline, and Shell transferred valuable trademarks and brands to low-tax locations.
internalization of such spillovers is crucial for the success of MNCs (Roberts, 2005), and prior literature identifies transfer prices as a means to achieve spillover internalization (Bouwens et al., 2017).

Nevertheless, most of the existing research regarding international transfer pricing for intangibles neglects spillovers (e.g., Johnson, 2006; De Waegenaere et al., 2012; Juranek et al., 2018). This omission is problematic, because an MNC’s tax-minimization incentives may interfere with the internalization of spillovers and, thus, affect the optimal location of an intangible and the corresponding transfer prices for its use. Therefore, the neglect to address spillovers restricts scholars’ and tax regulators’ understanding of MNCs’ incentives regarding the location of intangibles. In this study, we examine how spillovers affect an MNC’s choice of an intangible’s location and the corresponding transfer price for the use of this intangible.

We study a decentralized MNC with a headquarters and a domestic division in a high-tax country. A foreign division operates in a low-tax country. The MNC has a ready-to-use intangible (for example, a brand or a trademark), which both divisions use to generate profits, and the MNC sets a royalty-based transfer price for the use of this intangible. Our model comprises three decisions. First, the headquarters determines the intangible’s location. Second, the headquarters sets the royalty rate that determines the royalty-based transfer price (hereafter, ‘royalty payment’). The non-owning division makes the royalty payment. Third, the divisions choose marketing activities to increase their expected profits from using the intangible.

When the headquarters determines the intangible’s location, it chooses among four location possibilities: the domestic division (hereafter, ‘domestic ownership’), the foreign division (hereafter, ‘foreign ownership’), both divisions jointly (hereafter, ‘joint ownership’), or the headquarters (hereafter, ‘headquarters ownership’) can own the intangible.

For joint ownership, we study a stylized setting where neither division makes a royalty payment. The reason for this stylized setting is to clearly separate joint ownership from domestic, foreign, and headquarters ownership. For example, if the foreign division owns a higher proportion of the intangible under joint ownership, the domestic division transfers a royalty payment to the foreign
division. The setting in which the foreign division owns the intangible reflects this payment scheme.

Throughout this paper, we take it as given that ownership of the intangible is identical with being the legal owner. Because the intangible already exists and needs no further development, we assume that the tax authorities accept the legal ownership for tax transfer pricing. Section 8 critically discusses the assumption that the MNC can arbitrarily choose the intangible’s owner and, thus, the location of the intangible.

After the location choice, the headquarters stipulates the royalty rate that requires the non-owning division to pay a prespecified percentage of its contribution margin so that the royalty payment is variable. In general, a royalty payment can be a lump sum, variable, or a combination of these two forms. According to the OECD transfer-pricing guidelines (OECD, 2017), MNCs can both use a lump sum or a variable royalty payment for intangibles. When the MNC uses a lump sum, the OECD transfer-pricing guidelines allow discretion to adjust the lump sum ex post. For example, in many instances, U.S. tax law requires MNCs that use a lump sum to adjust the royalty payment ex post if the actual revenues deviate substantially from the projected revenues (Johnson, 2006; Boos, 2003). An ex post adjustment transforms a lump sum into a variable payment (Johnson, 2006). Additionally, previous studies show that the majority of licensing agreements use a variable royalty payment (e.g., Bousquet et al., 1998; Juranek et al., 2018; Macho-Stadler et al., 1996; Vishwasrao, 2007). Therefore, we restrict our attention to a variable royalty payment.

After observing the royalty rate, the divisions engage in marketing activities such as advertising and providing comprehensive customer service to increase their expected profits (Roberts, 2005; Sandner and Block, 2011). Each division’s marketing activities create an additive spillover on the other division’s contribution margin.

Additive spillovers appear to be descriptive of many instances. For example, a successful domestic marketing campaign (using internet advertisements, product placements, and influencer pro-

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2 The legal owner, for example, protects the intangible from undesired usage by competitors and, if necessary, takes legal steps to discourage or stop competitors from using the intangible. However, we do not model this protection function performed by the intangible’s owner.
motion of the brand) may reach potential foreign customers that are not covered by the foreign marketing activities. When tourists are on holiday in a foreign country, they may purchase candies or sodas for which they had seen a domestic advertisement. A European person may buy clothes of a brand for which she or he has seen a product placement in an U.S. sitcom, although she or he has never been reached by the brand’s local European marketing activities. In sum, these examples suggest that spillovers occur as add-ons for other divisions’ contribution margin. Therefore, we incorporate additive spillovers into our model.

In our model, the royalty rate has two functions: first, paying for the use of the intangible to determine the expected after-tax profits and, second, affecting the divisions’ marketing activities with respect to spillover internalization. Each division perfectly internalizes the spillover the division creates on the other division’s contribution margin when the division chooses efficient marketing activities. The two functions of the royalty rate create potentially conflicting objectives for the MNC’s transfer pricing (e.g., Baldenius et al., 2004; Hyde and Choe, 2005; Labro, 2019). Our analysis shows that the headquarters trades off three conflicting objectives when it determines the intangible’s location and sets the royalty rate. These objectives are spillover internalization by the domestic division, spillover internalization by the foreign division, and tax minimization. Each of the MNC’s location possibilities generates a different outcome for the tradeoff of these objectives.

When the foreign division owns the intangible, the MNC legally shifts profit from the high to the low-tax country. Using the optimal royalty rate under foreign ownership, the foreign division’s marketing activities and, thus, the foreign division’s spillover internalization are more efficient than the domestic division’s marketing activities. In sum, for a low domestic division’s spillover, the MNC optimally locates the intangible in the low-tax country.

In contrast, because the headquarters does not set a royalty rate, the MNC does not shift profits under joint ownership. Thus joint ownership does not affect the MNC’s tax liability. Without affecting the divisions’ marketing activities and ultimately the divisions’ spillover internalization by setting a royalty rate, the MNC achieves more efficient domestic division’s marketing activities under joint ownership than under foreign ownership. The foreign division’s marketing activities

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are less efficient in this case than under foreign ownership. Under a medium domestic division spillover, none of the three objectives outweighs the others, and thus joint ownership is optimal.

Under domestic ownership, the MNC achieves the most efficient marketing activities and, thus, spillover internalization, from the domestic division. Moreover, the foreign division’s marketing activities are less efficient than under either foreign or joint ownership. Additionally, the MNC shifts profits from the low- to the high-tax country. Nevertheless, for a high domestic division’s spillover, its internalization is so important that the MNC optimally locates the intangible in the high-tax country.

In sum, our findings show that the tradeoff among tax minimization and spillover internalization by the two divisions may explain the ‘home bias’, i.e., the empirical evidence that MNCs tend to hold intangibles in the parent’s high-tax country (Karkinsky and Riedel, 2012; Griffith et al., 2014; Heckemeyer et al., 2018). The empirical studies proxy for an intangible’s location by the country in which the intangible is officially recognized for the first time. None of these studies considers potential relocations. Because the aim of our paper is to identify an additional economic explanation for the empirically documented ‘home bias’, we follow the interpretation of the empirical studies that the country of the intangible’s first official recognition is the intangible’s location.

By using the optimal location of an intangible, we show that variations in the tax regulation affect the divisions’ marketing activities and the MNC’s optimal location of the intangible in various ways. By defining the legal environment, governments and tax authorities determine MNCs’ abilities to engage in profit shifting. When tax regulators narrow the arm’s length range, they restrict MNCs’ profit-shifting possibilities. A narrower arm’s length range affects the tradeoff between the MNC’s objectives of spillover internalization by both divisions and tax minimization. Under domestic ownership, a narrower arm’s length range implies that, ceteris paribus, more profit is shifted from the low- to the high-tax country and thus that the ‘home bias’ decreases. Additionally, domestic marketing activities can decrease in this case. This is in line with prior research (Desai et al., 2006; Hong and Smart, 2010; Juranek et al., 2018). However, the divisions’ marketing activities might also increase or be unaffected. We show that whether a narrower arm’s length range
is detrimental to the divisions’ marketing activities depends on the size of the domestic division’s spillover. These findings represent an extension of the literature.

A higher tax rate differential increases the MNC’s benefits from shifting profits from the high- to the low-tax country. When the MNC uses an interior value of the arm’s length range as the royalty rate, the MNC reacts to the better profit shifting possibilities by inducing the domestic division to reduce its marketing activities and the foreign to increase its marketing activities. Additionally, a higher tax rate differential increases the costs of shifting profits from the low- to the high-tax country. Because a higher tax rate differential does not affect the benefits from spillover internalization, the range of the domestic division’s spillover for which the MNC optimally locates the intangible in the domestic division narrows. Thus the ‘home bias’ decreases.

In our main analysis, we assume that the MNC uses one royalty rate and, thus, keeps one set of books. Alternatively, the MNC could also keep two sets of books and use two separate royalty rates: an internal royalty rate for coordinating internal decisions—in our model, marketing activities—and a tax royalty rate for determining the MNC’s tax liability. Previous studies state that, in the case of a tax audit, many MNCs fear negative consequences from keeping two sets of books (see, e.g., Halperin and Srinidhi, 1991; Narayanan and Smith, 2000; Smith, 2002; Baldenius et al., 2004; Johnson, 2006). Thus many MNCs likely refrain from using two sets of books and use one set of books. Additionally, recent empirical evidence by Klassen et al. (2017) documents that the majority of U.S.-based MNCs keep one set of books. Therefore, we study a setting in which the MNC uses one royalty rate. However, we also examine a model with two sets of books. Our analysis shows that the MNC always locates the intangible in the foreign division when the MNC faces a one-sided spillover from the domestic division on the foreign division’s contribution margin. When both divisions generate a spillover, the ‘home bias’ might also occur when the MNC keeps two sets of books.4

3 We are not interested in the question of which factors influence the MNC’s decision to keep either one or two sets of books (see, e.g., Haak et al., 2017).

4 Section 7 analyzes the model with two sets of books.
The remainder of the paper proceeds as follows. The next two sections present the related literature and the model. Section 4 presents two benchmarks for the location of the intangible. Section 5 depicts the optimal location of the intangible. Section 6 provides comparative statics. Section 7 analyzes how keeping two sets of books affects the optimal location of the intangible. Section 8 concludes the paper.

2 Literature Review

The literature on the transfer pricing of intangibles (with tax considerations) is scarce. Johnson (2006) examines different transfer pricing methods for intangibles. She studies a setting in which two divisions consecutively create an intangible. Her results show that royalty-based transfer pricing with renegotiation leads to first-best investments when the investments are either quasi-independent or substitutes. Unlike our model, that of Johnson (2006) does not consider that spillovers are prevalent when several divisions simultaneously use the intangible. In contrast, we show that spillovers affect MNCs’ optimal royalty rates and a ‘home bias’ may occur when MNCs face a tradeoff between tax minimization and spillover internalization.

Our paper relates to De Simone and Sansing (2019). They investigate whether cost sharing arrangements serve to shift intangible-related profits to low-tax countries when spillovers exist. De Simone and Sansing (2019) show that if the spillover of the domestic division exceeds the foreign division’s spillover on domestic profits, a cost sharing arrangement can be useful to shift profits. This result occurs because the IRS ignores spillovers. By illustrating the impact of spillovers on an intangible’s optimal location, we complement the findings of De Simone and Sansing (2019). In contrast to De Simone and Sansing (2019), we examine the costs and benefits of the location choice of a ready-to-use intangible rather than its development.

De Waegenaere et al. (2012) model a patent race among MNCs that make research and development investments that potentially create an intangible. The MNC that wins the patent race exploits the intangible during the subsequent production of tangible assets. The production can take place either domestically or in the foreign country. This location choice affects the social welfare of the
De Waegenaere et al. (2012) show that a weaker enforcement of the arm’s length principle may improve social welfare. By showing that a narrower arm’s length range may decrease the activities of an MNC, our model adds to the findings of De Waegenaere et al. (2012). In addition, we show that, depending on the spillovers, the MNC’s marketing activities can also increase for a narrower arm’s length range.

Recently, Juranek et al. (2018) investigate how different transfer pricing methods that determine the arm’s length range affect MNCs’ investment decisions when the intangible is located in a low-tax country. Moreover, they examine how effectively a source tax curtails profit shifting via royalty payments. Unlike our model, that of Juranek et al. (2018) does not consider spillovers. We show that spillovers together with decreasing profit shifting possibilities affect MNCs’ marketing activities.

This paper is also related to the empirical literature on the location of intangibles. The empirical studies by Karkinsky and Riedel (2012) and Heckemeyer et al. (2018) find a large ‘home bias’ for intangibles. In line with this empirical evidence, our analysis shows that, for a high spillover of the domestic division on the foreign division’s contribution margin, the MNC optimally locates the intangible in the domestic division in the high-tax country.

In addition, the empirical literature on the location of intangibles (Dischinger and Riedel, 2011; Karkinsky and Riedel, 2012; Griffith et al., 2014; Dischinger et al., 2014; Heckemeyer et al., 2018) offers several potential explanations for ‘home bias’. These explanations are as follows: The headquarters often finances the development of the intangible and bears the risk, making the headquarters the legal owner of the intangible (Karkinsky and Riedel, 2012). Moreover, the headquarters realizes economies of scale in the administration and management process of intangibles (Karkinsky and Riedel, 2012; Heckemeyer et al., 2018), and headquarters managers value their influence over valuables and, thus, seek to keep them under the headquarters managers’ control (Dischinger et al., 2014). Furthermore, MNCs want to avoid taxes upon the repatriation of profits from the foreign division to the headquarters (Dischinger et al., 2014) and to minimize payments regarding withholding taxes (Griffith et al., 2014; Heckemeyer et al., 2018). Additionally, particularities
in tax transfer pricing regulation and the law that regulates and protects the intangible can make locating the intangible in the low-tax country unattractive (Heckemeyer et al., 2018).

Whereas these potential explanations for the ‘home bias’ mostly relate to particularities of tax regulations and law, we illustrate the impact of spillovers on the location of intangibles. Thus, we provide an additional economic explanation for the empirically documented ‘home bias’.

3 Model Description

We examine an MNC operating a foreign division $F$ located in a low-tax country, where its income is taxed at a rate $t$. Additionally, the MNC has a downstream division $D$ and a headquarters $HQ$, which are both in a high-tax country, where an income tax rate of $t + h$ prevails, with $0 \leq t, h \leq 1$ and $t + h \leq 1$. The parameter $h$ captures the tax rate differential between the low- and the high-tax country. We investigate whether the MNC optimally locates an intangible, which the divisions use, in the low- or in the high-tax country. We focus on the costs and benefits of the intangible’s location in the post-development stage, i.e., the MNC has a ready-to-use intangible, when the MNC can arbitrarily choose the location. Section 8 revisits this assumption.

At time $t = 0$, the headquarters locates the intangible to maximize the MNC’s overall after-tax profit $\Pi_{HQ}$. We consider four location possibilities for the intangible: the domestic division, the foreign division, both divisions jointly, or the headquarters can own the intangible. We label the four location possibilities as domestic ownership, foreign ownership, joint ownership, and headquarters ownership.

As we consider administered royalty rates in a one-set-of-books setting for all location possibilities, at time $t = 1$, the headquarters stipulates a single royalty rate for the use of the intangible by the division that does not own it. We examine a royalty rate because it is common in tax reporting related to intangibles (Johnson, 2006). The headquarters applies the royalty rate for internal decision making and for calculating the tax liabilities, so the royalty rate must comply with tax law. To ensure compliance, the headquarters chooses a royalty rate from an arm’s length range. Therefore, the royalty rate $\gamma$ belongs to the range of arm’s length royalty rates $\left[\gamma, \overline{\gamma}\right]$ with $0 < \gamma < \overline{\gamma} < 1$, where
the boundaries of the arm’s length range are exogenous (see, e.g., Baldenius et al., 2004; Johnson, 2006).

Based on prior research, we tie the royalty rate $\gamma \in \mathbb{R}$ to the non-owning division’s contribution margin (e.g., Johnson, 2006; Bornemann, 2018). The ex ante determined royalty rate requires the non-owning division $i$ to pay a prespecified percentage of its realized contribution margin $M_i$, $i = F, D$ to the owning party. We refer to this realized payment as the royalty payment.

At time $t = 2$, while considering the intangible’s location and the royalty rate, both divisions affect the contribution margins by choosing their marketing activities to maximize the after-tax profit of their own division. Division $i$’s contribution margin comprises a direct contribution margin, a spillover from the other division’s marketing activities, and an additional component. First, the use of the intangible generates a direct contribution margin $x^i_L$. Without loss of generality, we normalize $x^i_L$ to zero. Each division can engage in marketing activities $\theta_i$ that increase the direct contribution margin to $x^i_H$ with probability $\theta_i$. In other words, $P(x_i = x^i_H | \theta_i) = \theta_i$ and $P(x_i = x^i_L | \theta_i) = 1 - \theta_i$.

We normalize $x^i_H$ to one. For example, marketing activities may increase the popularity of a brand name, leading to higher sales and, ceteris paribus, ultimately to a higher contribution margin. The marketing activities are costly. We capture the height of the marketing costs by $k > 0$ so that the costs resulting from marketing activities $\theta_i$ are $c_i = \frac{k}{2} \theta_i^2$. We assume that $k$ is sufficiently large to ensure that $0 \leq \theta_i \leq 1$ holds.

The marketing costs are tax deductible and, therefore, reduce the divisions’ taxable income. Although the marketing costs are tax deductible the headquarters cannot verify the marketing costs

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5 Our assumptions that the MNC is decentralized and that the divisions maximize the profit of their own division appear descriptive (Baldenius et al., 2004) and are widely used in the transfer pricing literature (e.g., Johnson, 2006; Baldenius and Reichelstein, 2006; Göx and Schiller, 2007; Johnson et al., 2018).

6 The divisions’ marketing activities create two additional intangibles, one in each division. We label the additional intangibles as shared domestic brand value and shared foreign brand value. Given the model setup, the domestic (foreign) division owns the shared domestic (foreign) brand value. The foreign (domestic) division makes a royalty payment for the shared domestic (foreign) brand value to the domestic (foreign) division. To keep the focus on the main research question, i.e., how the presence of marketing activities with spillovers affect the MNC’s decision regarding the location and royalty rate for the ready-to-use intangible, we normalize the royalty rate for the shared domestic (foreign) brand value to zero. We thank an anonymous reviewer for providing this insight.
related to that particular intangible. The reason is that the divisions have additional tax-deductible costs that can be lumped together with the marketing costs, so the headquarters cannot infer which part of the tax-deductible costs relates to the particular marketing activities. For example, the headquarters can observe the total cost of employing a marketing team, but the headquarters might not be able to verify the costs of specific marketing activities.

Second, division $j$’s marketing activities generate a spillover on the contribution margin of division $i$. While one division’s marketing activities, which use the intangible, do not affect the intangible’s availability to the other division, the simultaneous use of the intangible for both divisions’ marketing activities is, nevertheless, not independent. We model such interdependencies with a linear spillover $\beta_j$, with $0 \leq \beta_j \leq 1$. Division $i$ experiences a spillover on the contribution margin when the marketing activities of division $j$ are successful. Division $j$’s marketing activities are successful when division $j$ generates a high direct contribution margin $x_{jH}$. Therefore, division $j$’s expected spillover on division $i$’s contribution margin is $\theta_j \beta_j$.\(^7\) We label $\beta_D$ ($\beta_F$) as the domestic (foreign) division’s spillover.

Negative spillovers might also occur. However, if the spillover is sufficiently detrimental, it is reasonable that the division that imposes negative externalities on the other division is not allowed to engage in marketing activities related to the intangible. A typical example of negative spillovers are luxury brands. Other divisions’ marketing activities can be detrimental when the divisions inflate the market and, thus, destroy the exclusiveness of the brand. For example, Burberry recently attracted adverse media attention by burning tons of clothes rather than discounting them and, therefore, possibly inflating the market.\(^8\) This example shows that MNCs attempt to prevent negative expected externalities. Thus, we assume nonnegative spillovers.

Third, with regard to the contribution margin, division $i$ generates an additional component $\varepsilon_i$ with $E[\varepsilon_i] = 0$, and $\varepsilon_D$ and $\varepsilon_F$ are independent and identically distributed. In sum, division $i$’s

\(^7\) The realized spillover might be larger than the division’s realized direct contribution margin.

expected contribution margin from using the intangible is \( E[M_i] = \theta_i + \theta_j \beta_j + E[\epsilon_i] \).

At time \( t = 3 \), the divisions’ contribution margins are realized so that the specified royalty rate at \( t=1 \) determines the actual royalty payment(s). Correspondingly, the divisions’ after-tax profits \( \Pi_D \) and \( \Pi_F \), and thus, the MNC’s overall after-tax profit \( \Pi_{HQ} \) is realized.\(^9\)

Figure 1 shows the timing of the game.

![Figure 1: Timeline](image)

### 4 Benchmarks for the Location of the Intangible

This section provides two benchmarks for the analysis of the location of an intangible when spillovers exist. The first benchmark examines the location of an intangible when the headquarters can verify and dictate the marketing activities, i.e., the first-best solution. The second benchmark considers the location of an intangible in a no-tax world.

#### 4.1 Benchmark 1: First-Best Solution

In this benchmark, the headquarters can verify the marketing activities and thus enforces the first-best marketing activities.

When the foreign division owns the intangible, the domestic division makes a royalty payment to the foreign division in the low-tax country. Consequently, with foreign ownership, the MNC legally shifts profits from the high- to the low-tax country. This profit shifting reduces the MNC’s

\(^9\) The divisions’ after-tax profits \( \Pi_D \) and \( \Pi_F \) and the MNC’s overall after-tax profit \( \Pi_{HQ} \) depend on the location choice and the royalty rate. Whenever necessary, we present the expected profit functions in the main text or in the appendix.
tax liability so that, ceteris paribus, the expected MNC’s overall after-tax profit increases. When the headquarters sets the royalty rate as high as possible, i.e., $\gamma = \bar{\gamma}$, profit shifting to the foreign division in the low-tax country is maximal. Because the headquarters dictates the marketing activities for each possible location of the intangible and the MNC obtains profit shifting benefits only under foreign ownership, the MNC optimally locates the intangible in the foreign division.

Given the royalty rate $\bar{\gamma}$, the first-best marketing activities from the headquarters’ perspective are as follows:\(^\text{10}\)

$$
\theta_{D,fb} = \frac{1}{k} + \frac{\beta_D (1 - t) + h \bar{\gamma}}{k(1 - t - h)},
$$

$$
\theta_{F,fb} = \frac{1}{k} + \frac{\beta_F (1 - t - h + \bar{\gamma} h)}{k(1 - t)}.
$$

The first-best solution shows that the headquarters considers the spillover that each division creates on the contribution margin of the other division. Thus the domestic (foreign) division’s first-best marketing activities are increasing in the domestic (foreign) division’s spillover $\beta_D (\beta_F)$.

**Lemma 1.** *In the first-best solution, the MNC locates the intangible in the low-tax foreign division.*

4.2 **Benchmark 2: No-Tax World**

In the first benchmark, the headquarters was able to verify and dictate the marketing activities of the divisions. In the no-tax-world benchmark, as previously mentioned, the headquarters cannot verify the divisions’ marketing activities. Nevertheless, by setting the royalty rate, the headquarters can affect the divisions’ profits and, thus, their marketing activities.

Without taxes and the corresponding tax regulation that requires the royalty rate to belong to the arm’s length range, the headquarters is free to design the royalty rate to maximize the MNC’s overall expected profit, which is the sum of the divisions’ expected profits. For each possible location of the intangible, the royalty rate affects the divisions’ expected profits in a particular way. When the domestic (foreign) division owns the intangible, the foreign (domestic) division makes a royalty payment to the domestic (foreign) division. When the headquarters owns the intangible, 

\(^\text{10}\) See the appendix for all proofs.
both the domestic and the foreign division make a royalty payment to the headquarters. Under joint ownership, neither division makes a royalty payment. Given the location choice and the royalty rate \( \gamma \), each division chooses its marketing activities to maximize its own expected profit. For each possible location, table 1 depicts the marketing activities of both divisions.\(^{11}\) In addition, table 1 shows the divisions’ first-best marketing activities without taxes.

<table>
<thead>
<tr>
<th>Marketing activities by</th>
<th>Location choice</th>
<th>First-best solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>domestic ownership</td>
<td>foreign ownership</td>
</tr>
<tr>
<td>Domestic division</td>
<td>( \frac{1+\gamma \beta_D}{k} )</td>
<td>( \frac{1-\gamma}{k} )</td>
</tr>
<tr>
<td>Foreign division</td>
<td>( \frac{1-\gamma}{k} )</td>
<td>( \frac{1+\gamma \beta_F}{k} )</td>
</tr>
</tbody>
</table>

Table 1: Marketing activities chosen by the domestic and foreign divisions in the no-tax world.

Comparing the divisions’ marketing activities under the four possible locations with the no-tax first-best solution (see table 1) reveals that, in general, the headquarters cannot stipulate a single royalty rate \( \gamma \) that induces both divisions to choose the first-best marketing activities. Given our model setup, one exception to this statement exists. When the headquarters owns the intangible and the foreign division’s and the domestic division’s spillovers are equal, the headquarters can use the royalty rate \( \gamma = -\beta_D = -\beta_F \) to make the divisions choose the first-best marketing activities.

The reason that, in general, the headquarters cannot induce first-best marketing activities by both divisions is as follows. Each division’s successful marketing activities create a spillover on the contribution margin of the other division. When each division maximizes its own expected contribution margin less its marketing costs, i.e., \( \theta_i + \theta_j \beta_j + E[\varepsilon_i] - \frac{k}{k} \theta_i^2 \) with \( i, j = D, F \) and \( i \neq j \), the division does not internalize the spillover that it creates on the contribution margin of the other division. However, because the headquarters maximizes the MNC’s overall expected profit, the first-best marketing activities of a division reflect this spillover. By choosing an appropriate

\(^{11}\) See the appendix for all proofs and thresholds.
royalty rate, the headquarters can induce a division to internalize the spillover whenever the possible location requires a royalty payment. While maximizing the MNC’s overall expected profit requires that both divisions internalize their respective spillovers, the headquarters has only a single royalty rate that can induce the internalization of the spillovers. Consequently, in general, the headquarters cannot induce both divisions to internalize their spillovers and, thus, choose first-best marketing activities. Under joint ownership, no royalty payments occur, and thus, the headquarters cannot use the royalty rate to induce spillover internalization.

Under domestic, foreign, or headquarters ownership, the headquarters trades off the spillover internalization of the domestic and foreign divisions. Under domestic ownership, the headquarters stipulates the royalty rate \( \gamma^D_{NT} = \frac{\beta_D - \beta_F}{1 + \beta_D} \) to maximize the MNC’s overall expected profit.\(^\text{12}\) Compared to the first-best solution without taxes, this royalty rate induces both divisions to choose inefficiently low marketing activities. When the foreign division owns the intangible, the headquarters sets the royalty rate \( \gamma^F_{NT} = \frac{\beta_F - \beta_D}{1 + \beta_F} \), and again, both divisions implement inefficiently low marketing activities compared to the first-best solution without taxes. Under headquarters ownership, the headquarters stipulates the royalty rate \( \gamma^{HQ}_{NT} = -\frac{1}{2} (\beta_D + \beta_F) \). Therefore, the division that creates a lower spillover on the other division’s contribution margin chooses inefficiently high marketing activities, while the other division implements inefficiently low marketing activities. In sum, while the divisions’ marketing activities can be inefficiently high when the headquarters owns the intangible, both divisions choose inefficiently low marketing activities when either of the two divisions owns the intangible.

In the no-tax world, joint ownership is never optimal. The reason is that with either of the three other location possibilities, the headquarters can use the royalty rate to achieve better spillover internalization. When the domestic division’s spillover \( \beta_D \) is higher than the foreign division’s spillover \( \beta_F \), the MNC prefers foreign to domestic ownership and vice versa. The reason is as

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\(^{12}\) The subscript \( NT \) highlights the no-tax cases, whereas the absence of the \( NT \) subscript signals tax-world considerations. The superscript indicates the owner of the intangible.
follows. Under foreign ownership and $\beta_D > \beta_F$, the headquarters sets a negative royalty rate $\gamma_{NT}^F$ to ensure reasonable spillover internalization in the domestic division. This spillover internalization is particularly important because the domestic division’s spillover is higher than the foreign division’s spillover. When the headquarters owns the intangible, the headquarters ensures the spillover internalization with a negative royalty rate, and thus both divisions receive a royalty payment. When the difference between the two divisions’ spillovers is small, both divisions’ spillover internalization is more efficient under headquarters ownership than under domestic or foreign ownership. Lemma 2 summarizes these findings.

**Lemma 2.** In the no-tax world, for $\beta_D > \beta_F$ and

- for a large difference in the divisions’ spillovers, i.e., $\beta_F < 2 - \sqrt{3}$ and $\beta_D > \beta_{HQ,NT}^F$, the MNC locates the intangible in the foreign division;

- for a small difference in the divisions’ spillovers, the MNC locates the intangible at the headquarters.

In the no-tax world, for $\beta_F \geq \beta_D$ and

- for a large difference in the divisions’ spillovers, i.e., $\beta_D < 2 - \sqrt{3}$ and $\beta_F > \beta_{HQ,NT}^D$, the MNC locates the intangible in the domestic division;

- for a small difference in the divisions’ spillovers, the MNC locates the intangible at the headquarters.

Despite the absence of marginal costs, the optimal royalty rate for each optimal location of the intangible is negative.

## 5 Location of the Intangible

This section (1) presents both divisions’ marketing activities and the optimal royalty rates under foreign and domestic ownership, (2) discusses both divisions’ marketing activities under joint and headquarters ownership, and (3) shows that, depending on the size of the domestic division’s spillover, the headquarters optimally chooses domestic, foreign, or joint ownership.

By locating the intangible in the foreign division, as the first-best solution benchmark in section 4.1 shows, the MNC reduces its tax liability. Thus the MNC’s expected overall after-tax profit under
foreign ownership contains a tax-saving position and is as follows:

\[
E \left[ \Pi_{HQ}^F \right] = (1 - t - h) \left[ \theta_D + \theta_F^D \beta_F^D \frac{k}{2} \theta_D^2 \right] + (1 - t) \left[ \theta_F + \theta_D \beta_D - \frac{k}{2} \theta_F^2 \right] + h \gamma (\theta_D + \theta_F \beta_F) . \tag{1}
\]

The divisions’ optimal marketing activities under foreign ownership and taxes are:13

\[
\theta_F^D = \frac{1}{k} (1 - \gamma) , \tag{2}
\]

\[
\theta_F^F = \frac{1 + \gamma \beta_F}{k} . \tag{3}
\]

Comparing these divisions’ marketing activities with the divisions’ marketing activities in the no-tax-world benchmark in section 4.2 shows that taxes do not affect the divisions’ incentives regarding the marketing activities. While taxes do not affect the divisions’ incentives to engage in marketing activities, the headquarters incorporates the taxes and, thus, seeks to induce the first-best marketing activities, which we present in section 4.1.

Beyond the divisions’ spillovers, with taxes, the headquarters also considers the MNC’s overall tax liability. As the no-tax-world benchmark in section 4.2 shows, the headquarters trades off the spillover internalization of both divisions. The optimal royalty rate in the no-tax benchmark reflects this tradeoff. As tax regulation requires the royalty rate to belong to the arm’s length range \([\gamma, \bar{\gamma}]\), the headquarters cannot use a royalty rate smaller than \(\gamma\) when taxes exist. While spillover internalization by both divisions under foreign ownership requires a negative royalty rate, tax regulation dictates a positive royalty rate from the arm’s length range. Beyond the tradeoff between spillover internalization in the foreign and domestic divisions, the headquarters’ tax minimization objective creates an additional tradeoff. The royalty rate that optimally trades off the three conflicting

\[\text{See the appendix for all threshold values and findings in this section.}\]
objectives under foreign ownership is as follows:

$$
\gamma^F = \begin{cases} 
\bar{\gamma} & \text{for } \beta_D \leq \beta_{D,1}^F \\
\frac{\beta_D^2(1-t-h)+h(1+\beta_F)-\beta_D(1-t)}{1-t+h+\beta_F^2(1-t-2h)} & \text{for } \beta_D \in \left(\beta_{D,1}^F, \beta_{D,2}^F\right) \\
\gamma & \text{for } \beta_D \geq \beta_{D,2}^F.
\end{cases}
$$

(4)

Under foreign ownership and a very low domestic division’s spillover, i.e., for $$\beta_D \leq \beta_{D,1}^F$$, the headquarters uses the highest possible royalty rate $$\bar{\gamma}$$ to minimize the MNC’s tax liability. If the domestic division’s spillover is higher, i.e., $$\beta_D \in \left(\beta_{D,1}^F, \beta_{D,2}^F\right)$$, its internalization is more important for the headquarters, which thus then no longer applies a corner solution of the arm’s length range. Instead, the headquarters uses a royalty rate that is lower than the highest possible royalty rate $$\bar{\gamma}$$, i.e., the headquarters uses $$\frac{\beta_D^2(1-t-h)+h(1+\beta_F)-\beta_D(1-t)}{1-t+h+\beta_F^2(1-t-2h)}$$. The royalty rate approaches the lower bound of the arm’s length range, $$\gamma$$, for a spillover $$\beta_D \geq \beta_{D,2}^F$$. In other words, by partially forgoing the profit-shifting benefits, the headquarters alleviates the inefficiencies from the domestic division’s spillover internalization. Nevertheless, from the headquarters’ perspective, both divisions still choose inefficiently low marketing activities.

When the domestic division owns the intangible, the MNC’s expected overall after-tax profit contains a term reflecting that the domestic division pays taxes on incoming royalty payments. Thus the MNC’s expected overall after-tax profit with domestic ownership is as follows:

$$
E \left[ \Pi_{HQ}^D \right] = (1-t-h) \left[ \theta_D + \theta_F \beta_F - \frac{k}{2} \theta_D^2 \right] + (1-t) \left[ \theta_F + \theta_D \beta_D - \frac{k}{2} \theta_F^2 \right] - h\gamma(\theta_F + \theta_D \beta_D).
$$

(5)

The divisions’ optimal marketing activities under domestic ownership and taxes are:

$$
\theta_D^D = \frac{1}{k} (1 + \gamma \beta_D),
$$

(6)

$$
\theta_F^D = \frac{1}{k} (1 - \gamma).
$$

(7)

Comparing these divisions’ marketing activities with the divisions’ marketing activities in the no-tax-world benchmark in section 4.2 shows that taxes do not affect the divisions’ incentives to engage in marketing activities. This finding is similar to the finding under foreign ownership and
taxes. While taxes do not affect the divisions’ incentives to engage in marketing activities, the headquarters incorporates the taxes and, thus, seeks to induce the first-best marketing activities, which we present in section 4.1.

As under taxes and foreign ownership, the divisions’ spillover internalization and tax minimization are conflicting objectives and affect the headquarters’ choice of the royalty rate under domestic ownership. When the domestic division owns the intangible, the headquarters optimally trades off the three conflicting objectives by using the following royalty rate:

\[
\gamma^D = \begin{cases} 
\gamma & \text{for } \beta_D \leq \beta_{D,1}^D \\
\frac{(1-t)\beta_D^2 - h(1+\beta_D)-(1-t-h)\beta_F}{\beta^2(1-t+h)+1-t-2h} & \text{for } \beta_D \in \left(\beta_{D,1}^D, \beta_{D,2}^D\right) \\
\bar{\gamma} & \text{for } \beta_D \geq \beta_{D,2}^D.
\end{cases}
\]  

(8)

For a low domestic division’s spillover \(\beta_D \leq \beta_{D,1}^D\), the headquarters uses the royalty rate \(\gamma\) that shifts the lowest possible profit to the domestic division in the high-tax country. When the domestic division’s spillover is higher, i.e., \(\beta_D \in \left(\beta_{D,1}^D, \beta_{D,2}^D\right)\), its internalization is more important for the MNC. By using a higher royalty rate, the headquarters can improve the domestic division’s spillover internalization. Consequently, by setting a royalty rate higher than \(\bar{\gamma}\), i.e.,

\[
\frac{(1-t)\beta_D^2 - h(1+\beta_D)-(1-t-h)\beta_F}{\beta^2(1-t+h)+1-t-2h},
\]

the headquarters shifts more profit to the domestic division in the high-tax country. This royalty rate approaches the upper bound of the arm’s length range, \(\bar{\gamma}\), for a spillover \(\beta_D \geq \beta_{D,2}^D\). In other words, by accepting more profit shifting to the high-tax country, the headquarters alleviates the inefficiencies from the domestic division’s spillover internalization. Nevertheless, from the headquarters’ perspective, both divisions still choose inefficiently low marketing activities.

As previously mentioned, under joint ownership, the headquarters applies no royalty rate by definition. Thus the headquarters cannot use the royalty rate to induce the divisions to internalize their spillovers in their marketing activities. Each division’s marketing activities are \(1/k\). These marketing activities are the same as those under joint ownership in the no-tax-world benchmark, and again, these marketing activities are inefficiently low (see table 1). Because the headquarters cannot stipulate a royalty rate for the use of the intangible, the headquarters shifts profit neither to
the foreign division in the low-tax country nor to the domestic division in the high-tax country.

When the headquarters owns the intangible, each division’s marketing activities are \( \frac{1 - \gamma}{k} \). Again, these marketing activities are the same as those under headquarters ownership in the no-tax-world benchmark. The no-tax-world benchmark states that the headquarters uses a negative royalty rate when the headquarters owns the intangible and no tax regulation requires the royalty rate to belong to a certain arm’s length range. Nevertheless, given the arm’s length range, the headquarters cannot set a negative royalty rate. In addition, because both divisions, in expectation, make a royalty payment to the headquarters, the MNC shifts profits from the foreign division in the low-tax country to the headquarters in the high-tax country. This profit shifting increases the MNC’s tax liability.

With headquarters ownership, each division’s marketing activities are lower than those under joint ownership. Consequently, with headquarters ownership, the divisions’ spillover internalization is less efficient than that under joint ownership. The reason is that the royalty rate has to belong to the arm’s length range, which prevents the headquarters from providing the divisions with appropriate incentives for spillover internalization. Moreover, while the headquarters does not shift profits under joint ownership, the headquarters considers profit shifting to the high-tax country and, thus, a higher tax liability for the MNC under headquarters ownership. In sum, both the divisions’ spillover internalization and tax minimization are less efficient under headquarters ownership than under joint ownership. Thus with taxes, headquarters ownership is never optimal.

In the following, we illustrate that the headquarters optimally chooses one of the three remaining location possibilities, i.e., foreign, domestic, or joint ownership. The optimal location of the intangible depends on both divisions’ spillovers. Hereafter, we describe the optimal location of the intangible in terms of the size of the domestic division’s spillover. The thresholds for the domestic division’s spillover depend on the foreign division’s spillover.

For a low domestic division’s spillover, i.e., \( \beta_D < \beta_f^F \) with \( \beta_f^F > \beta_{D,2}^F \), the headquarters op-

\[14\text{ The superscript F and the subscript J indicate that the threshold } \beta_f^F \text{ is the level of the domestic division’s spillover at which the headquarters is indifferent between foreign and joint ownership.} \]
timally locates the intangible in the foreign division, and the MNC shifts profits to the low-tax country. As previously mentioned, both divisions implement inefficiently low marketing activities relative to the first-best solution. The inefficiency with regard to the domestic division’s marketing activities is small for a low domestic division’s spillover, so the objective of tax minimization drives the MNC’s location choice for the intangible. Thus as in the first-best solution, the tax minimization objective determines the location choice for a low domestic division’s spillover.

Under joint ownership, by definition, the MNC cannot shift profits using a royalty rate. By shifting no profit to the low-tax country, the MNC faces a higher tax liability than under foreign ownership. Moreover, under joint ownership, the foreign division’s marketing activities are less efficient than under foreign ownership. Nevertheless, joint ownership is optimal for a medium domestic division’s spillover. The reason is that the domestic division’s marketing activities here are more efficient than those under foreign ownership. Because the headquarters necessarily uses a royalty rate that belongs to the arm’s length range, the domestic division’s marketing activities $\frac{1-\gamma}{k}$ under foreign ownership are lower than those under joint ownership $\frac{1}{k}$. Thus, while the domestic division’s marketing activities are still inefficiently low under joint ownership, the headquarters mitigates this inefficiency relative to the domestic division’s inefficient spillover internalization under foreign ownership. In sum, because the domestic division’s marketing activities are less inefficient under joint ownership than under foreign ownership, the headquarters optimally chooses joint ownership for medium spillover.

If the domestic division’s spillover is high, the inefficient marketing activities of the domestic division are especially detrimental, meaning that the domestic division’s spillover internalization drives the location choice. By locating the intangible in the domestic division, the headquarters substantially mitigates the inefficient spillover internalization of the domestic division. Compared to foreign and joint ownership, the domestic division distorts its marketing activities less downwards. Beyond the more efficient domestic spillover internalization, the headquarters considers the negative effect on the spillover internalization of the foreign division and the negative consequences resulting from profit shifting to the low-tax country. First, the royalty rate induces the foreign divi-
sion to implement lower marketing activities than under foreign or joint ownership. As previously mentioned, the foreign division’s marketing activities under foreign or joint ownership are already inefficiently low. Second, the headquarters accepts profit shifting to the domestic division in the high-tax country, and thus, the MNC has a higher tax liability. In sum, the analysis shows that, for a high domestic division spillover, i.e., $\beta_D \geq \beta_{DJ}$, $\beta_{DJ} < \beta_{DJ}$, the positive effect of the domestic division’s spillover internalization outweighs the negative effects, and thus domestic ownership is optimal.15 In other words, a ‘home bias’ occurs.

Proposition 1 summarizes the findings. Figure 2 illustrates the intangible’s optimal location and the corresponding royalty rates, and figure 3 depicts an example of the optimal location of the intangible.

Proposition 1. For $h < \frac{1-t}{2}, \gamma < \frac{1-t}{1-t+h}$,16 $\beta_F < \overline{\beta}_F$ with $0 < \overline{\beta}_F$, and

- a low domestic division’s spillover, i.e., $\beta_D \leq \beta_F^*$, the MNC locates the intangible in the foreign division in the low-tax country;
- a medium domestic division’s spillover, i.e., $\beta_F^* < \beta_D < \beta_{DF}$, the MNC chooses joint ownership for the intangible;
- a high domestic division’s spillover, i.e., $\beta_D \geq \beta_{DF}$, the MNC locates the intangible in the domestic division in the high-tax country.

6 Effects of Variations in Tax Regulation on Marketing Activities and the Location of Intangibles

This section examines how variations in tax regulation affect the divisions’ marketing activities and the MNC’s location choice for the intangible. We use the optimal location of an intangible and the divisions’ corresponding marketing activities from proposition 1. For a low domestic division’s

15 The superscript D and the subscript J indicate that the threshold $\beta_{DJ}$ is the level of the domestic division’s spillover at which the headquarters is indifferent between domestic and joint ownership.
16 The first threshold is decreasing in the tax rate $t$. For $t = 0.1$ or $t = 0.3$, the first threshold allows a maximal tax rate differential of 0.45 or 0.35, respectively. The second threshold is decreasing in $t$ and $h$. For $t = 0.3$ and $h = 0.25$, the maximal upper bound of the arm’s length range $\gamma$ is 0.737. Numerous tax jurisdictions fulfill these two criteria regarding $h$ and $\gamma$. 

22
spillover, $\theta^D_F$ and $\theta^F_F$ from equations (2) and (3) represent the domestic and the foreign division’s marketing activities. For a medium domestic division’s spillover, each division’s marketing activities are $1/k$. For a high domestic division’s spillover, $\theta^D_D$ and $\theta^D_F$ from equations (6) and (7) represent the domestic and the foreign division’s marketing activities. The two threshold values for the domestic division’s spillover $\beta^F_D$ and $\beta^D_D$ determine the location of the intangible. The regulatory parameters of interest are the lower and the upper bound of the arm’s length range and the tax rate differential.

When the lower bound of the arm’s length range $\gamma$ increases and the upper bound $\bar{\gamma}$ decreases, the arm’s length range narrows. By narrowing the arm’s length range, tax regulators affect the MNC’s profit-shifting possibilities and, thus, the divisions’ marketing activities and the MNC’s location choice for the intangible. For $\beta_D \leq \beta^F_D$, the foreign division owns the intangible and the headquarters stipulates the highest possible royalty rate $\gamma = \bar{\gamma}$. In this case, a narrower arm’s length range, i.e., a decreasing upper bound of the arm’s length range, reduces the royalty payment that the domestic division expects to pay for using the intangible. Because the domestic division trades off the expected revenues and costs related to the intangible, the reduced expected royalty payment increases the domestic division’s marketing activities. Moreover, as the foreign division receives a
lower expected royalty payment, its expected profit decreases. The foreign division reacts to this decrease by reducing its marketing activities. The analysis shows that the increase in the domestic division’s marketing activities outweighs the decrease in the foreign division’s marketing activities.

For $\beta_D \geq \beta_{D,2}^D$, the domestic division optimally owns the intangible, and the headquarters stipulates the highest possible royalty rate $\gamma = \overline{\gamma}$. By retaining more expected profit in the foreign division, a narrower arm’s length range, i.e., a decreasing upper bound of the arm’s length range $\gamma$, reduces the domestic division’s marketing activities. Because the foreign division trades off the expected profit and costs related to the intangible, this higher expected profit in the foreign division increases the foreign division’s marketing activities. This positive effect on the foreign division’s marketing activities overcompensates for the negative effect on the domestic division’s marketing activities such that, in sum, the MNC’s overall marketing activities increase for a decreasing upper bound of the arm’s length range.

For $\beta_D \in [\beta_{D,2}^F, \beta_{F}^F]$, the MNC optimally locates the intangible in the foreign division, and the
headquarters stipulates the lowest possible royalty rate $\gamma = \gamma$. Because an increase in the lower bound of the arm’s length range $\gamma$ makes the use of the intangible more costly for the domestic division, its marketing activities decrease. While the domestic division’s marketing activities are already inefficiently low under foreign ownership, greater inefficiencies in the domestic division’s marketing activities result from an increasing $\gamma$. Moreover, an increase in the lower bound of the arm’s length range $\gamma$ increases the royalty payment that the foreign division expects to receive from the domestic division. The higher royalty payment induces the foreign division to more efficiently internalize its spillover such that the foreign division increases its marketing activities. The negative effect of the domestic division’s marketing activities dominates the positive effect of the foreign division’s marketing activities. Thus the MNC’s overall marketing activities decrease for an increase in the lower bound of the arm’s length range.

For $\beta_D \in [\beta_{D,1}^{F,1}, \beta_{D,1}^{F,2}]$, the domestic division optimally owns the intangible, and the headquarters stipulates the lowest possible royalty rate, i.e., $\gamma = \gamma$. In this case, a higher lower bound of the arm’s length range $\gamma$ increases the royalty payment that the foreign division expects to pay under domestic ownership. The higher expected royalty payment increases the domestic division’s expected profit. The domestic division trades off the expected profit and the costs related to the intangible such that the marketing activities of the domestic division increase. Because the domestic division’s marketing activities are inefficiently low under domestic ownership, the domestic division’s marketing activities become more efficient for an increasing lower bound of the arm’s length range $\gamma$. Nevertheless, in response to the higher expected royalty payment, the foreign division implements less efficient marketing activities such that the MNC’s overall marketing activities decrease for an increasing lower bound of the arm’s length range.

For $\beta_D \in (\beta_{D,1}^{F,1}, \beta_{D,1}^{F,2})$ and $\beta_D \in (\beta_{D,1}^{D,1}, \beta_{D,1}^{D,2})$, the MNC locates the intangible in the foreign and domestic division, respectively, and the headquarters uses an interior value of the arm’s length range as the royalty rate. Thus a narrower arm’s length range does not affect the divisions’ marketing activities.

For $\beta_D \in (\beta_{D,1}^{F}, \beta_{D,1}^{D})$, the MNC optimally chooses joint ownership (see proposition 1). Because,
by definition, the headquarters cannot use the royalty rate to affect the divisions’ marketing activities when both divisions jointly own the intangible, a varying arm’s length range does not affect the divisions’ marketing activities. Proposition 2 summarizes the findings regarding the MNC’s marketing activities and a narrower arm’s length range.

**Proposition 2.** For $\beta_D \leq \beta_{D,1}^F$ or $\beta_D \geq \beta_{D,2}^D$, a narrower arm’s length range leads to more marketing activities for the MNC.

For $\beta_D \in \left[\beta_{D,2}^F, \beta_{D,1}^F\right]$ or $\beta_D \in \left[\beta_{D,1}^D, \beta_{D,2}^D\right]$, a narrower arm’s length range leads to less marketing activities for the MNC.

For any other domestic division’s spillover, i.e., $\beta_D \in \left[\beta_{D,1}^F, \beta_{D,2}^F\right]$, $\beta_D \in \left[\beta_{D,1}^D, \beta_{D,2}^D\right]$, or $\beta_D \in \left[\beta_{D,2}^D, \beta_{D,1}^D\right]$, a narrower arm’s length range does not affect the marketing activities of the MNC.

A narrower arm’s length range affects the optimal location of the intangible. Under domestic and foreign ownership, a narrower arm’s length range limits the headquarters’ possibilities to optimally trade off the conflicting objectives of spillover internalization by both divisions and tax minimization. This reduced flexibility decreases the benefits from domestic and foreign ownership. Consequently, the threshold level $\beta_{F}^D$ decreases while the threshold level $\beta_{D}^F$ increases for a narrower arm’s length range. In other words, the ranges of the domestic division’s spillover $\beta_D$ for which the MNC locates the intangible in the foreign or domestic division also narrow for a narrower arm’s length range. Thus a narrower arm’s length range decreases the ‘home bias’. Proposition 3 summarizes this finding.

**Proposition 3.** A narrower arm’s length range decreases the threshold level $\beta_{F}^D$ and increases the threshold level $\beta_{D}^F$, meaning that the ‘home bias’ decreases.

In the following, we study the effect of an increasing tax rate differential on the divisions’ marketing activities and the MNC’s optimal location choice for the intangible.

As previously mentioned, taxes and, consequently, the tax rate differential do not affect the divisions’ incentives regarding marketing activities. However, the headquarters incorporates the tax rate differential. By using the royalty rate, the headquarters seeks to induce first-best marketing activities, which change given an increasing tax rate differential.

Under foreign or domestic ownership, an increase in the tax rate differential $h$ affects the divisions’ marketing activities only when the headquarters stipulates a royalty rate that is an interior
value of the arm’s length range. A higher tax rate differential increases the MNC’s benefits from shifting profits from the high- to the low-tax country but does not affect the MNC’s benefits from spillover internalization. The MNC reacts to the higher tax rate differential by increasing (decreasing) the royalty rate under foreign (domestic) ownership. A higher paid (lower received) royalty rate induces the domestic division to reduce its marketing activities under foreign (domestic) ownership. Due to the lower royalty rate, under domestic ownership, the foreign division retains a higher share of its expected profits. Because the foreign division trades off its expected profit and the costs of the use of the intangible, the foreign division’s marketing activities increase. Under foreign ownership, due to the higher royalty rate, the MNC shifts more profit from the domestic to the foreign division such that the foreign division’s expected profit increases. Again, the foreign division increases its marketing activities in response to its higher expected profit.

Because, by definition, the headquarters does not use a royalty rate when both divisions jointly own the intangible, an increasing tax rate differential does not affect the divisions’ marketing activities.

**Proposition 4.** An increasing tax rate differential 
- decreases or does not affect the domestic division’s marketing activities and
- increases or does not affect the foreign division’s marketing activities.

While an increasing tax rate differential does not affect the MNC’s benefits from spillover internalization, a higher tax rate differential makes profit shifting from the low- to the high-tax country especially costly for the MNC. Thus an increasing tax rate differential affects the headquarters’ tax-minimization objective. The three optimal location choices of proposition 1 address this objective in varying ways. While the headquarters legally reduces the MNC’s tax liability under foreign ownership, it shifts profits from the low- to the high-tax country and, thus, accepts a higher tax liability under domestic ownership. Under joint ownership, the MNC does not shift profits at all. Consequently, a higher tax rate differential increases both the thresholds $\beta_F^J$ and $\beta_D^J$, and the range of the domestic division’s spillover $\beta_D$ for which the MNC locates the intangible in the foreign
(domestic) division broadens (narrow). Thus a higher tax rate differential decreases the ‘home bias’. Proposition 5 summarizes this finding.

**Proposition 5.** An increasing tax rate differential \( h \) increases the threshold levels \( \beta_F^J \) and \( \beta_D^J \), meaning that the ‘home bias’ decreases.

7 Discussion of Location of the Intangible with Two Sets of Books

Thus far in this paper, we have assumed that the MNC keeps one set of books. This assumption reflects the empirical evidence of Klassen et al. (2017), who find that most U.S.-based MNCs keep one set of books. However, the MNC might also keep two sets of books. In other words, rather than using one royalty rate \( \gamma \) for both internal decision-making—in our model, the marketing activities—and calculating the tax liability, the headquarters stipulates two different royalty rates. The internal royalty rate \( \gamma_i \) coordinates the internal decisions, and the tax royalty rate \( \gamma_r \) determines the divisions’ and, thus, the MNC’s overall tax liability. While the headquarters is free to design the internal royalty rate, the tax royalty rate has to belong to the arm’s length range. By using different royalty rates for tax minimization and spillover internalization, the MNC mitigates the conflict among the different objectives. Given the arm’s length range, the headquarters stipulates the tax royalty rate to minimize the MNC’s tax liability.

This section discusses how keeping two sets of books instead of one set of books affects the optimal location of the intangible. First, we analyze the MNC’s optimal location choice for the intangible and the corresponding royalty rates when the MNC faces a one-sided spillover from the domestic division’s marketing activities on the foreign division’s contribution margin, i.e., \( \beta_F = 0 \). Second, we discuss how keeping two sets of books affects the MNC’s location choice for the intangible with two-sided spillovers, i.e., \( \beta_D, \beta_F > 0 \).

With \( \beta_F = 0 \) and two sets of books, the divisions’ after-tax profits under foreign ownership are
as follows:  

\[
E \left[ \Pi_{D}^{F,TSB} \right] = (1 - t - h) \left[ \theta_D - \frac{k}{2} \theta_D^2 \right] - \gamma_i \theta_D + \gamma_r \theta_D (t + h)
\]

and

\[
E \left[ \Pi_{F}^{F,TSB} \right] = (1 - t) \left[ \theta_F + \beta_D \theta_D - \frac{k}{2} \theta_F^2 \right] + \gamma_i \theta_D - t \gamma_r \theta_D.
\]

To minimize tax liability, the headquarters uses the highest possible tax royalty rate, i.e., \( \gamma_r = \overline{\gamma} \), under foreign ownership. Thus, for \( \gamma_r = \overline{\gamma} \), equation (1), which we introduced in section 5, states the expected MNC’s overall after-tax profit.

In the following, we show that by locating the intangible in the foreign division, the MNC replicates the first-best solution when the MNC keeps two sets of books and faces a one-sided spillover. Under foreign ownership, the divisions’ marketing activities are as follows:

\[
\theta_{F,N}^{F,TSB} = \frac{1}{k} + \frac{\overline{\gamma} (t + h) - \gamma_i}{k (1 - t - h)}
\]  

(9)

and

\[
\theta_{F}^{F,TSB} = \frac{1}{k}.
\]  

(10)

By setting the following internal royalty rate, the headquarters induces the domestic division to perfectly internalize its spillover:

\[
\gamma_i^F = \overline{\gamma} - (1 - t) \beta_D.
\]  

(11)

Because the internal royalty rate \( \gamma_i^F \) does not affect the foreign division’s marketing activities, the headquarters achieves the first-best marketing activities by both divisions.

Thus, by keeping two sets of books and under foreign ownership, the headquarters resolves the spillover internalization problem for the one-sided domestic division’s spillover. In addition, the headquarters achieves optimal tax minimization by legally shifting the highest possible profit from the high- to the low-tax country.

\[\text{The superscript TSB denotes the MNC’s use of two sets of books.}\]

\[\text{All proofs are in the appendix.}\]
Under two-sided spillovers and two sets of books, the headquarters sets, as under the one-sided spillover, the tax royalty rate to minimize the MNC’s tax liability and determines the internal royalty rate to induce both divisions to internalize their spillovers. Again, under foreign ownership, the MNC shifts profits from the high- to the low-tax country. Therefore, also under two-sided spillovers and two sets of books, locating the intangible in the foreign division allows the headquarters to optimally meet the tax minimization objective.

Beyond tax minimization, the headquarters also considers the objective of spillover internalization in both divisions. The no-tax-world benchmark in section 4.2 shows that the headquarters faces a tradeoff between inducing the domestic division and the foreign division to internalize their spillovers. When the MNC faces taxes and keeps two sets of books, this tradeoff persists for the internal royalty rate. Lemma 2 shows that without taxes, depending on the size of the divisions’ spillovers, the MNC optimally trades off spillover internalization in the two divisions by locating the intangible in the foreign division, in the domestic division, or at the headquarters. The findings of lemma 2 indicate that foreign ownership does not necessarily optimally address the tradeoff among the three objectives of tax minimization and spillover internalization by the two divisions. Consequently, for two-sided spillovers and the inherent tradeoff regarding spillover internalization in the two divisions, the MNC does not always choose foreign ownership for the intangible when the MNC keeps two sets of books. In other words, a ‘home bias’ might occur when the MNC keeps two sets of books and faces two-sided spillovers.

8 Conclusion

This paper examines the optimal location of an intangible and the corresponding royalty rate for using the intangible when the MNC trades off tax minimization and spillover internalization. Each division perfectly internalizes the spillover the division creates on the other division’s contribution margin when the division chooses efficient marketing activities.

Our analysis shows that for a low domestic division’s spillover, the MNC locates the intangible in the foreign division that operates in the low-tax country. Because under foreign ownership the
domestic division in the high-tax country makes a royalty payment to the foreign division, this location of the intangible allows the MNC to optimally shift profits. At the same time, the domestic division’s marketing activities are inefficiently low.

For a medium domestic division’s spillover, the MNC mitigates the inefficiency in the domestic division’s marketing activities with joint ownership. The foreign division engages in less efficient marketing activities, and the MNC shifts no profit to the low-tax country. Nevertheless, the benefit from more efficient marketing activities in the domestic division outweigh the less efficient ones in the foreign division and the higher tax liability such that joint ownership is optimal.

For a high domestic division’s spillover, internalization is especially important, so the MNC particularly benefits from efficient marketing activities in the domestic division. By locating the intangible in the domestic division, the MNC can induce more efficient marketing activities in the domestic division. Beyond the benefits from the more efficient marketing activities, the MNC also incurs two negative effects when the domestic division owns the intangible. First, the foreign division implements less efficient marketing activities than under foreign or joint ownership. Second, by shifting profits from the low- to the high-tax country, the MNC increases its tax liability. For a high domestic division’s spillover, the benefits resulting from better spillover internalization in the domestic division outweigh these negative effects of domestic ownership. Thus, the MNC locates the intangible in the domestic division that operates in the high-tax country. This finding provides a potential explanation for the empirically documented ‘home bias’. In other words, the objective of spillover internalization may induce MNCs to locate their intangibles in the domestic high-tax countries.

Although our main example describes intangibles such as trademarks and brands, our results also apply to patents. For example, after launching a patented drug, the MNC needs marketing activities to ensure consumer awareness of the active ingredient and, thus, increase the contribution margin of the patent. These marketing activities may also have a spillover on the contribution margin of divisions located in other countries that also sell the patented drug, such as through publications in renowned medical journals and word-of-mouth promotion.
The previously described findings state that the MNC locates the intangible in the domestic division in the high-tax country given a high domestic division’s spillover. Patents relate to a specific product, technology, or process, whereas trademarks likely affect the entire business of an MNC. Therefore, trademarks seem to have higher spillovers than patents, meaning that our model predicts that trademarks exhibit a larger ‘home bias’ than patents. Empirical evidence supports this implication. Whereas Heckemeyer et al. (2018) document that 95.3 percent of U.S. trademarks are located in the U.S., Karkinsky and Riedel (2012) depict that only 57.1 percent of patent applications occur in the parent country. In sum, according to our model, the difference in the ‘home bias’ of trademarks and patents might partially stem from spillovers.

In addition, our model shows that a narrower arm’s length range affects the MNC’s marketing activities and the location of the intangible. Depending on the optimal location of the intangible, the MNC’s marketing activities may increase, decrease, or be unaffected. Because a narrower arm’s length range limits the MNC’s ability to optimally trade off the conflicting objectives of spillover internalization by both divisions and tax minimization, a narrower arm’s length range decreases the ‘home bias’.

A higher tax rate differential also decreases the ‘home bias’. As the costs for shifting profits from the low- to the high-tax country increase for a higher tax rate differential, the MNC reduces the range of the domestic division’s spillover for which the MNC locates the intangible in the domestic division. Additionally, when the MNC uses an interior value of the arm’s length range as the royalty rate, the MNC reacts to a higher tax rate differential by inducing the domestic division to decrease its marketing activities and inducing the foreign division to increase its marketing activities.

Throughout the paper, we assume that the MNC has a ready-to-use intangible and that the headquarters can arbitrarily decide where to locate it. This assumption is often not without loss of generality. First, when developing or purchasing an intangible, the MNC incurs tax deductible (multi-period) development or amortization costs. Second, tax authorities usually consider the development stage of an intangible and levy exit taxes. Both tax deductible costs from developing or purchasing an intangible and exit taxes potentially affect the optimal location of the intangible.
Beyond spillover internalization, the (present value of the) tax deductibility benefit of the development or purchasing costs affects the MNC’s optimal location of the intangible. Thus, if we introduce development or purchasing costs for the intangible into the model, they will affect the thresholds of the domestic division’s spillover that determine the optimal location of the intangible.

When the MNC transfers a valuable ready-to-use intangible, this transfer triggers exit taxes. MNCs are aware of exit taxes and incorporate them when choosing the intangible’s location. Nevertheless, the impact of exit taxes depends on the features of the intangible’s development. Either any of the divisions might be able to develop the intangible or the development of the intangible requires expertise or environments that are only available in one of the two countries. When either of the two divisions is able to develop the intangible, the MNC anticipates the overall costs and benefits, such as exit taxes, of the intangible’s location and consequently develops the intangible in the preferred division.

When the development of the intangible needs to take place in one of the two countries, the costs and benefits of the post-development stage and exit taxes affect the location choice. Consequently, if we introduce exit taxes into the model, they might affect the threshold levels of the domestic division’s spillover that determine the optimal location of the intangible. However, consider, for example, an intangible that the MNC optimally locates in the domestic high-tax country in the post-development stage and additionally needs development expertise solely available in the domestic high-tax country. In this case, exit taxes do not affect the MNC’s location choice. The MNC always wants to locate the intangible in the domestic high-tax country. In sum, while multiple factors, such as exit taxes and tax deductible costs from developing or purchasing the intangible, may affect the MNC’s location choice for an intangible, we illustrate that spillover internalization is also a relevant factor.


9 Appendix

9.1 Proof of Lemma 1

Under foreign ownership, the MNC’s expected overall after-tax profit is as follows:

\[
E[\Pi_{HQ}] = (1 - t - h) \left[ \theta_D + \beta_F \theta_F - \frac{k}{2} \theta_D^2 \right] + (1 - t) \left[ \theta_F + \beta_D \theta_D - \frac{k}{2} \theta_F^2 \right] + h \gamma [\theta_D + \beta_F \theta_F].
\]

To minimize the MNC’s tax liability, the headquarters chooses the highest possible tax royalty rate \( \gamma \). The headquarters chooses the marketing activities to maximize the MNC’s expected overall after-tax profit. The headquarters’ first (FOCs) and second order conditions (SOCs) are as follows:

\[
FOC \theta_D : (1 - t - h)(1 - k \theta_D) + \beta_D (1 - t) + h \gamma = 0
\]

\[
SOC \theta_D : (1 - t - h)(-k) < 0
\]

\[
FOC \theta_F : (1 - t - h) \beta_F + (1 - t)(1 - k \theta_F) + \gamma h \beta_F = 0
\]

\[
SOC \theta_F : (1 - t)(-k) < 0.
\]

Thus the FOCs determine a local maximum: \( \theta_{D,fb} \) and \( \theta_{F,fb} \).

9.2 Proof of Lemma 2

For \( t = 0 \) and \( h = 0 \), \( \theta_{D,fb} \) and \( \theta_{F,fb} \) determine the first-best marketing activities in the no-tax world. These marketing activities are as follows:

\[
\theta_{D,fb,NT} = \frac{1 + \beta_D}{k},
\]

\[
\theta_{F,fb,NT} = \frac{1 + \beta_F}{k}.
\]

First, we calculate the marketing activities and the royalty rate(s) for all location possibilities. Second, we compare the MNC’s expected overall profits for the location possibilities to determine the optimal location of the intangible in the no-tax world.

9.2.1 Domestic Ownership

The divisions’ expected profits are as follows:

\[
E[\Pi_D] = \theta_D + \beta_F \theta_F - \frac{k}{2} \theta_D^2 + \gamma (\theta_F + \beta_D \theta_D),
\]

\[
E[\Pi_F] = (1 - \gamma) (\theta_F + \beta_D \theta_D) - \frac{k}{2} \theta_F^2.
\]
The divisions choose their marketing activities to maximize their divisions’ expected profit:

\begin{align*}
\text{FOC}_D : & \quad 1 - k\theta_D + \gamma \beta_D = 0 \\
\text{SOC}_D : & \quad -k < 0 \\
\text{FOC}_F : & \quad 1 - k\theta_F - \gamma = 0 \\
\text{SOC}_F : & \quad -k < 0.
\end{align*}

Thus the FOCs determine a local maximum:

\begin{align*}
\theta_{D,NT}^D &= \frac{1 + \gamma \beta_D}{k}, \\
\theta_{F,NT}^D &= \frac{1 - \gamma}{k}.
\end{align*}

The MNC’s expected overall profit is as follows:

\[ E[\Pi_{HQ,NT}] = \theta_D(1 + \beta_D) + \theta_F(1 + \beta_F) - \frac{k}{2}(\theta_D^2 + \theta_F^2). \]

Given \( \gamma \), the headquarters cannot achieve first-best marketing activities. Given the divisions’ marketing activities, the headquarters chooses the royalty rate to maximize the MNC’s expected overall profit:

\begin{align*}
\text{FOC}_\gamma : & \quad \frac{1}{k} \left[ \beta_D^2(1 - \gamma) - \gamma - \beta_F \right] \\
\text{SOC}_\gamma : & \quad \frac{1}{k} \left( \beta_D^2 + 1 \right) \left( -\frac{1}{2} \right) < 0.
\end{align*}

Thus the FOC determines a local maximum. The optimal royalty rate is as follows:

\[ \gamma_{NT}^D = \frac{\beta_D^2 - \beta_F}{1 + \beta_D^2}. \]

Because \( \gamma_{NT}^D < 1 \), \( \theta_{D,NT}^D (\gamma_{NT}^D) < \theta_{D,fb,NT} \) and \( \theta_{F,NT}^D (\gamma_{NT}^D) < \theta_{F,fb,NT} \).

\subsection*{9.2.2 Foreign Ownership}

The divisions’ expected profits are as follows:

\begin{align*}
E[\Pi_D] &= (1 - \gamma)(\theta_D + \beta_F \theta_F) - \frac{k}{2}\theta_D^2, \\
E[\Pi_F] &= (\theta_F + \beta_D \theta_D) - \frac{k}{2}\theta_F^2 + \gamma(\theta_D + \beta_F \theta_F).
\end{align*}

The MNC’s expected overall profit remains in all no-tax scenarios unchanged and is given by
equation (14). The procedure of section 9.2.1 yields:

\[
\theta_{D,NT}^F = \frac{1 - \gamma}{k}, \quad (16)
\]

\[
\theta_{F,NT}^F = \frac{1 + \gamma \beta_F}{k}, \quad (17)
\]

\[
\gamma_{NT}^F = \frac{\beta_F^2 - \beta_D}{1 + \beta_F^2}.
\]

Because \( \gamma_{NT}^F < 1 \), \( \theta_{D,NT}^F (\gamma_{NT}^F) < \theta_{D,fb,NT} \) and \( \theta_{F,NT}^F (\gamma_{NT}^F) < \theta_{F,fb,NT} \).

### 9.2.3 Joint Ownership

The divisions’ expected profits are as follows:

\[
E[\Pi_D] = \theta_D + \beta_F \theta_F - \frac{k}{2} \theta_D^2,
\]

\[
E[\Pi_F] = \theta_F + \beta_D \theta_D - \frac{k}{2} \theta_F^2.
\]

The procedure of section 9.2.1 yields:

\[
\theta_{J,NT}^D = \theta_{J,NT}^F = \frac{1}{k}, \quad (18)
\]

where \( \theta_{J,NT}^D < \theta_{D,fb,NT} \) and \( \theta_{J,NT}^F < \theta_{F,fb,NT} \).

### 9.2.4 Headquarters Ownership

In expectation, both divisions make royalty payments for the intangible’s use. The divisions’ expected profits are as follows:

\[
E[\Pi_D] = (1 - \gamma)(\theta_D + \beta_F \theta_F) - \frac{k}{2} \theta_D^2,
\]

\[
E[\Pi_F] = (1 - \gamma)(\theta_F + \beta_D \theta_D) - \frac{k}{2} \theta_F^2.
\]

The procedure of section 9.2.1 yields that \( \theta_{D,NT}^{HQ} = \frac{1 - \gamma}{k} \), which is equal to (16), and \( \theta_{F,NT}^{HQ} = \frac{1 - \gamma}{k} \), which is equal to (13), and \( \gamma_{NT}^{HQ} = -\frac{1}{2} (\beta_D + \beta_F) \).

For \( \beta_D > \beta_D (\beta_D < \beta_F) \), \( \theta_{D,NT}^{HQ} (\gamma_{NT}^{HQ}) < (>) \theta_{D,fb,NT} \) and \( \theta_{F,NT}^{HQ} (\gamma_{NT}^{HQ}) > (<) \theta_{F,fb,NT} \).

### 9.2.5 Optimal Location of the Intangible in the No-Tax World

With \( \gamma = 0 \), the marketing activities under foreign ownership replicate the marketing activities under joint ownership. Because the optimal \( \gamma_{NT}^F \neq 0 \), \( E[\Pi_{HQ,NT}^F] > E[\Pi_{HQ,NT}^F] \). Thus foreign ownership strictly dominates joint ownership.
The comparison of the MNC’s expected overall profits under domestic and foreign ownership yields:

\[
E \left[ \Pi^D_{HQ,NT} \right] - E \left[ \Pi^F_{HQ,NT} \right] = \frac{\beta_D + \beta_F + \beta_D \beta_F (2 + \beta_D + \beta_F)}{2k(1 + \beta_D^2)(1 + \beta_F^2)} (\beta_D \beta_F - 1) < 0 \quad (> 0 \text{ for } \beta_D > (<) \beta_F)
\]

so that the headquarters prefers domestic to foreign ownership for \( \beta_D < \beta_F \) and vice versa.

For \( \beta_D < (>) \beta_F \), we compare the MNC’s expected overall profits under domestic (foreign) and headquarters ownership in the following. For \( i, j = F, D, \) and \( i \neq j \), this comparison yields:

\[
E \left[ \Pi^i_{HQ,NT} \right] - E \left[ \Pi^{HQ}_{HQ,NT} \right] = \frac{1 + \beta_i}{4k(1 + \beta_i^2)} \left[ (1 - \beta_i) \beta_i^2 - (1 - \beta_i) \beta_i^2 - 2k \beta_i (1 + \beta_i) \right]
\]

\[
> 0 \iff \beta_i < 2 - \sqrt{3} \text{ and } \beta_j > \frac{\beta_i (1 + \beta_i)}{1 - \beta_i} + \sqrt{2\beta_i} \sqrt{1 + \beta_i^2} =: \beta^{HQ}_{j,NT}.
\]

### 9.3 Proof of Proposition 1

Following the procedure of the no-tax world, we calculate the marketing activities and royalty rate(s), and compare the MNC’s expected overall after-tax profit for each location possibility to determine the optimal location of the intangible in the tax world.

#### 9.3.1 Domestic Ownership

The expected after-tax profits of the divisions are as follows:

\[
E \left[ \Pi^D_D \right] = (1 - t - h) \left[ \theta_D + \theta_F \beta_F + \gamma(\theta_F + \theta_D \beta_D) - \frac{k}{2} \theta_D^2 \right],
\]

\[
E \left[ \Pi^D_F \right] = (1 - t) \left[ (1 - \gamma)(\theta_F + \theta_D \beta_D) - \frac{k}{2} \theta_F^2 \right].
\]

The divisions choose their marketing activities to maximize their own after-tax profits:

\[
FOC \theta_D : (1 - t - h) \left( 1 + \gamma \beta_D - k \theta_D \right) = 0
\]

\[
SOC \theta_D : (1 - t - h) \left( -k \right) < 0
\]

\[
FOC \theta_F : (1 - t) \left( 1 - \gamma - k \theta_F \right) = 0
\]

\[
SOC \theta_F : (1 - t) \left( -k \right) < 0.
\]

Thus the FOCs determine a local maximum and the marketing activities are \( \theta^D_D = \frac{1}{k} (1 + \gamma \beta_D) \), which equals (12) and \( \theta^D_F = \frac{1}{k} (1 - \gamma) \), which equals (13). The MNC’s expected overall after-tax
The expected after-tax profit is:

\[
E \left[ \Pi_{HQ}^D \right] = (1 - t - h) \left[ \theta_D^P + \theta_F^P \beta_D - \frac{k}{2} (\theta_D^P) \right] \\
+ (1 - t) \left[ \theta_F^P + \theta_D^P \beta_D - \frac{k}{2} (\theta_D^P) \right] - h \gamma (\theta_F^P + \theta_D^P \beta_D). \tag{19}
\]

The headquarters chooses \( \gamma \) to maximize the MNC’s expected overall after-tax profit:

\[
FOC \gamma: \frac{1}{k} \left[ -\gamma (\beta_D^2 (1 - t + h) + 1 - t - 2h) + (1 - t) \beta_D^2 - h (1 + \beta_D) - (1 - t - h) \beta_F \right] = 0
\]

\[
SOC \gamma: -\frac{1}{k} \beta_D^2 (1 - t + h) + 1 - t - 2h.
\]

The SOC is negative for \( h < (1 - t)/2 \). Thus the FOC determines a local maximum and the optimal royalty rate is as follows:

\[
\gamma^D = \frac{(1 - t) \beta_D^2 - h (1 + \beta_D) - (1 - t - h) \beta_F}{\beta_D^2 (1 - t + h) + 1 - t - 2h}. \tag{20}
\]

The royalty rate needs to belong to the arm’s length range. For \( \gamma \leq \frac{1 - t}{1 - t + h} \) and \( h < \frac{1 - t}{2} \), \( \gamma^D > \gamma \) if and only if \( \beta_D > \beta_{D,1} \), where

\[
\beta_{D,1} := \frac{h + \sqrt{h^2 + 4 (1 - t - \gamma (1 - t + h))(h + (1 - t - h) \beta_F + (1 - t - 2h) \gamma)}}{2(1 - t - \gamma (1 - t + h))}.
\]

For \( \gamma \leq \frac{1 - t}{1 - t + h} \) and \( h < \frac{1 - t}{2} \), \( \gamma^D < \gamma \) if and only if \( \beta_D < \beta_{D,2} \), where

\[
\beta_{D,2} := \frac{h + \sqrt{h^2 + 4 (1 - t - \gamma (1 - t + h))(h + (1 - t - h) \beta_F + (1 - t - 2h) \gamma)}}{2(1 - t - \gamma (1 - t + h))},
\]

and \( \beta_{D,1} < \beta_{D,2} \). In sum, for \( \gamma \leq \frac{1 - t}{1 - t + h} \) and \( h < \frac{1 - t}{2} \) the optimal royalty rate is as stated in equation (8).

The domestic division’s marketing activities \( \theta_D^P (\gamma^D) \) equal its first-best activities \( \theta_{D,fb} \) if and only if \( \gamma = \frac{\beta_D (1 - t + h) \gamma}{\beta_D (1 - t - h)} \). \( \gamma \) is smaller than \( \frac{\beta_D (1 - t + h) \gamma}{\beta_D (1 - t - h)} \) so that \( \theta_D^P (\gamma^D) < \theta_{D,fb} \). Because \( \gamma^D > 0 \), \( \theta_F^P (\gamma^D) < \theta_{F,fb} \).

### 9.3.2 Foreign Ownership

The expected after-tax profits of the domestic and the foreign division are as follows:

\[
E \left[ \Pi_F^D \right] = (1 - t - h) \left[ (1 - \gamma) (\theta_D + \theta_F \beta_D) - \frac{k}{2} \theta_D^2 \right]
\]

\[
E \left[ \Pi_F^F \right] = (1 - t) \left[ \theta_F + \theta_D \beta_D + \gamma (\theta_D + \theta_F \beta_D) - \frac{k}{2} \theta_F^2 \right].
\]
The procedure of section 9.3.1 yields that \( \theta_F^F = \frac{1}{k}(1 - \gamma) \), which equals (16), and \( \theta_F^F = \frac{1 + \gamma \beta_F}{k} \), which equals (17). The MNC’s expected overall after-tax profit is as follows:

\[
E \left[ \Pi_{HQ}^F \right] = (1 - t - h) \left[ \theta_D^F + \theta_F^F \beta_F - \frac{k}{2} \left( \theta_F^F \right)^2 \right] + (1 - t) \left[ \theta_D^F + \theta_F^F \beta_D - \frac{k}{2} \left( \theta_F^D \right)^2 \right] + h \gamma \left[ \theta_D^F + \theta_F^F \beta_F \right].
\]

(21)

The headquarters chooses \( \gamma \) to maximize the MNC’s expected overall after-tax profit:

\[
FOC \gamma: \frac{1}{k} \left[ -\gamma (1 - t + h + \beta_F^2 (1 - t - 2h)) + \beta_F^2 (1 - t - h) + h (1 + \beta_F) - (1 - t) \beta_D \right] = 0
\]

\[
SOC \gamma: \frac{1}{k} \left( -1 + t - h + \beta_F^2 (-1 + 2h + t) \right).
\]

Because \( \beta_F \leq 1 \), the SOC is less or equal to \( \frac{\beta_F^2}{k} \left[ -1 + t - h - 1 + 2h + t \right] = \frac{\beta_F^2}{k} \left[ -2 + 2t + h \right] \), which is negative. Thus the FOC determines a local maximum and the optimal royalty rate is:

\[
\gamma_F^F = \frac{\beta_F^2 (1 - t - h) + h (1 + \beta_F) - \beta_D (1 - t)}{1 - t + h + \beta_F^2 (1 - t - 2h)}.
\]

(22)

The royalty rate needs to belong to the arm’s length range. \( \gamma_F^F > \gamma \) if and only if

\[
\beta_D < \beta_{D,2}^F := \frac{h - \gamma (1 - t + h) + \beta_F h + \beta_F^2 (1 - t - h) - \gamma (1 - t - 2h)}{1 - t}.
\]

\( \gamma_1^F < \gamma \) if and only if

\[
\beta_D > \beta_{D,1}^F := \frac{h - \gamma (1 - t + h) + \beta_F h + \beta_F^2 (1 - t - h) - \gamma (1 - t - 2h)}{1 - t},
\]

where \( \beta_{D,1}^F < \beta_{D,2}^F \). In sum, the optimal royalty rate is as stated in equation (4).

Because \( \gamma_F^F > 0 \), \( \theta_D^F (\gamma_F^F) < \theta_{D,fb} \). The foreign division’s marketing activities \( \theta_F^F (\gamma_F^F) \) equal its first-best activities if and only if \( \gamma = \frac{1 - t - h + \theta_F^F}{1 - t} \), which is larger than \( \gamma \). Thus \( \theta_F^F (\gamma_F^F) < \theta_{F,fb} \).

### 9.3.3 Joint Ownership

The divisions’ expected after-tax profits are as follows:

\[
E \left[ \Pi_D^J \right] = (1 - t - h) \left[ \theta_D + \theta_F \beta_F - \frac{k}{2} \theta_D^2 \right],
\]

\[
E \left[ \Pi_F^J \right] = (1 - t) \left[ \theta_F + \theta_D \beta_D - \frac{k}{2} \theta_F^2 \right].
\]
The procedure of section 9.3.1 yields that $\theta_D^H = \theta_F^H = \frac{1}{k}$, which equals (18). The MNC’s expected overall after-tax profit is as follows:

$$E \left[ \Pi_{HQ}^H \right] = (1 - t - h) \left[ \theta_D^H + \theta_F^H \beta_F - \frac{k}{2} (\theta_D^H)^2 \right] + (1 - t) \left[ \theta_F^H + \theta_D^H \beta_D - \frac{k}{2} (\theta_F^H)^2 \right].$$

(23)

### 9.3.4 Headquarters Ownership

The divisions’ expected after-tax profits are as follows:

$$E \left[ \Pi_D^{HQ} \right] = (1 - t - h) \left[ (1 - \gamma) (\theta_D + \theta_F \beta_F) - \frac{k}{2} \theta_D^2 \right],$$

$$E \left[ \Pi_F^{HQ} \right] = (1 - t) \left[ (1 - \gamma) (\theta_F + \theta_D \beta_D) - \frac{k}{2} \theta_F^2 \right].$$

The procedure of section 9.3.1 yields that $\theta_D^{HQ} = \frac{1}{k} (1 - \gamma)$, which equals (16), and $\theta_F^{HQ} = \frac{1}{k} (1 - \gamma)$, which equals (13). The MNC’s expected overall after-tax profit is as follows:

$$E \left[ \Pi_{HQ}^{HQ} \right] = (1 - t - h) \left[ \theta_D^{HQ} + \theta_F^{HQ} \beta_F - \frac{k}{2} \left( \theta_F^{HQ} \right)^2 \right]$$

$$+ (1 - t) \left[ (\theta_F^{HQ} + \theta_D^{HQ} \beta_D) - \frac{k}{2} \left( \theta_F^{HQ} \right)^2 \right] - h \gamma \left( \theta_F^{HQ} + \theta_D^{HQ} \beta_D \right).$$

(24)

The headquarters chooses $\gamma$ to maximize the MNC’s expected overall after-tax profit: $\gamma$: 

$$\text{FOC}\gamma: \frac{1}{k} \left[ \gamma \left( h(3 + 2\beta_D) - 2(1 - t) \right) - \beta_D (1 - t + h) - \beta_F (1 - t - h) + h \right],$$

$$\text{SOC}\gamma: \frac{1}{k} \left[ h(3 + 2\beta_D) - 2(1 - t) \right],$$

which is negative (positive) for $h < (>) \frac{2(1 - t)}{3 + 2\beta_D}$. For $h < \frac{2(1 - t)}{3 + 2\beta_D}$, the FOC determines a local maximum:

$$\gamma_1^{HQ} = \frac{1}{h(3 + 2\beta_D) - 2(1 - t)} \left[ \beta_D (1 - t + h) + \beta_F (1 - t - h) + h \right] < 0.$$

Because $E \left[ \Pi_{HQ}^{HQ} \right]$ is concave and the local maximizer is negative, the global maximizer is $\gamma$.

For $h > \frac{2(1 - t)}{3 + 2\beta_D}$, the FOC determines a local minimum:

$$\gamma_2^{HQ} = \frac{1}{h(3 + 2\beta_D) - 2(1 - t)} \left[ \beta_D (1 - t + h) + \beta_F (1 - t - h) + h \right],$$

which is larger than 1. Because $E \left[ \Pi_{HQ}^{HQ} \right]$ is convex and the local minimizer is larger than 1, the global maximizer is $\gamma$.  

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Because $\gamma > 0$, $\theta_{D}^{HQ}(\gamma) < \theta_{D,fb}$ and $\theta_{F}^{HQ}(\gamma) < \theta_{F,fb}$.

9.3.5 Optimal Location of the Intangible in the Tax World

The MNC’s expected overall after-tax profit under joint and headquarters ownership is given by equations (23) and (24), respectively. $E[\Pi_{HQ}^{D}] \geq E[\Pi_{HQ}^{F}]$ so that joint ownership dominates headquarters’ ownership.

In the following, we first compare the MNC’s expected overall after-tax profits under domestic and joint ownership. Second, we compare the MNC’s expected overall after-tax profits under foreign and joint ownership. Third, we show that, for $\beta_{F} < \bar{\beta}_{F}$, the first two steps fully describe the MNC’s optimal location of the intangible.

First, when $\beta_{D} \leq \beta_{D,1}^{D}$, equation (19) with $\gamma = \gamma$ determines the MNC’s expected overall after-tax profit under domestic ownership so that

$$E[\Pi_{HQ}^{D}] - E[\Pi_{HQ}^{I}] = \frac{\gamma}{2k} \left[ \beta_{D}^{2}[2(1-t) - \gamma(1-t+h)] - \beta_{D}2h - 2\beta_{F}(1-t-h) - \gamma(1-t-2h) \right],$$

which is quadratic in $\beta_{D}$ and negative for $\beta_{D} = 0$. Thus $E[\Pi_{HQ}^{D}] - E[\Pi_{HQ}^{I}] \geq 0$ if and only if $\beta_{D} \geq \beta_{D}^{I}$, where

$$\beta_{D}^{I} := \frac{h + \sqrt{h^{2} + \left[2(1-t) - \gamma(1-t+h)\right]2h + 2\beta_{F}(1-t-h) + \gamma(1-t-2h)}}{2(1-t) - \gamma(1-t+h)}, \quad (25)$$

with $0 < \beta_{D}^{I} < \beta_{D,1}^{D}$.

When $\beta_{D} \in \left(\beta_{D,1}^{D}, \beta_{D,2}^{D}\right)$, equation (19) with $\gamma = \frac{(1-t)\beta_{D}^{2}-h(1+\beta_{D})-(1-t-h)\beta_{F}}{\beta_{D}^{2}(1-t+h)+1-t-2h}$ determines the MNC’s expected overall after-tax profit under domestic ownership so that

$$E[\Pi_{HQ}^{D}] - E[\Pi_{HQ}^{I}] = \frac{1}{2k\left[1-t-2h+\beta_{D}^{2}(1-t+h)\right]} \left[ \beta_{F}(1-t-h) + h + \beta_{D}h - \beta_{D}^{2}(1-t) \right]^{2} > 0.$$

When $\beta_{D} \geq \beta_{D,2}^{D}$, equation (19) with $\gamma = \gamma$ determines the MNC’s expected overall after-tax profit under domestic ownership so that

$$E[\Pi_{HQ}^{D}] - E[\Pi_{HQ}^{I}] =$$

$$= \frac{\gamma}{2k} \left[ \beta_{D}^{2}[2(1-t) - \gamma(1-t+h)] - \beta_{D}2h - 2\beta_{F}(1-t-h) - \gamma(1-t-2h) \right],$$
which is quadratic in $\beta_D$ and negative for $\beta_D = 0$. Thus $E \left[ \Pi_{HQ}^D \right] - E \left[ \Pi_{HQ}^J \right] \geq 0$ if and only if

$$\beta_D \geq \frac{h + \sqrt{h^2 + [2(1-t) - \gamma(1-t+h)][2h + 2\beta_F(1-t-h) + \gamma(1-t-2h)]}}{2(1-t) - \gamma(1-t+h)},$$

which is smaller than $\beta_{D,2}^F$. Thus for $\beta_D \geq \beta_{D,2}^F$, the MNC chooses domestic ownership. In sum, for $\beta_D < (>) \beta_{D,1}^F$, joint (domestic) ownership dominates domestic (joint) ownership.

Second, when $\beta_D \leq \beta_{D,1}^F$, equation (21) with $\gamma = 7$ determines the MNC’s expected overall after-tax profit under foreign ownership so that

$$E \left[ \Pi_{HQ}^F \right] - E \left[ \Pi_{HQ}^J \right] =$$

$$= \frac{\gamma}{2k} \left[ 2h + 2h\beta_F - 2(1-t)\beta_D - \gamma(1-t+h) + \beta_F^2 \left( 2(1-t-h) - \gamma(1-t-2h) \right) \right]$$

$$\geq 0 \iff \beta_D \leq \frac{2h + 2h\beta_F - \gamma(1-t+h) + \beta_F^2 \left( 2(1-t-h) - \gamma(1-t-2h) \right)}{2(1-t)},$$

which is larger than $\beta_{D,1}^F$. Thus, for $\beta_D \leq \beta_{D,1}^F$, the MNC chooses foreign ownership.

When $\beta_D \in (\beta_{D,1}^F, \beta_{D,2}^F)$, equation (21) with $\gamma = \frac{\beta_F^2(1-t-h) + h(1+\beta_F - \beta_D(1-t))}{1-t+h+\beta_F^2(1-t-2h)}$ determines the MNC’s expected overall after-tax profit under foreign ownership so that

$$E \left[ \Pi_{HQ}^F \right] - E \left[ \Pi_{HQ}^J \right] =$$

$$= \frac{1}{2k \left[ 1 - t + h + \beta_F^2(1-t-2h) \right]} \left[ h - \beta_D(1-t) + \beta_F \left( h + \beta_F(1-t-h) \right) \right] > 0.$$

When $\beta_D \geq \beta_{D,2}^F$, equation (21) with $\gamma = 7$ determines the MNC’s expected overall after-tax profit under foreign ownership so that

$$E \left[ \Pi_{HQ}^F \right] - E \left[ \Pi_{HQ}^J \right] =$$

$$= \frac{\gamma}{2k} \left[ 2h + 2h\beta_F - 2(1-t)\beta_D - \gamma(1-t+h) + \beta_F^2 \left( 2(1-t-h) - \gamma(1-t-2h) \right) \right]$$

$$\geq 0 \iff \beta_D \leq \beta_D^F$$

with

$$\beta_D^F := \frac{2h + 2h\beta_F - \gamma(1-t+h) + \beta_F^2 \left( 2(1-t-h) - \gamma(1-t-2h) \right)}{2(1-t)},$$

(26)

$\beta_D^F$ which is larger than $\beta_{D,2}^F$. In sum, for $\beta_D < (>) \beta_D^F$, foreign (joint) ownership dominates joint (foreign) ownership.

Third, we compare the thresholds $\beta_D^F$ and $\beta_D^J$, which are both continuous in $\beta_F$. When $\beta_F = 0,$
\[ \beta^F_j < \beta^D_j \text{ for } h < \frac{1-t}{2} \text{ and } \gamma < \frac{1-t}{1-t+h}. \] Because

\[
\frac{d\beta^F_j}{d\beta_F} = \frac{1}{1-t} \left[ h + \beta_F \left[ 2(1-t-h) - \gamma(1-t-2h) \right] \right] > 0,
\]

\[
\frac{d^2\beta^F_j}{d(\beta_F)^2} = \frac{1}{1-t} \left[ 2(1-t-h) - \gamma(1-t-2h) \right] > 0,
\]

\( \beta^F_j \) is monotonically increasing and convex in \( \beta_F \).

Because

\[
\frac{d\beta^D_j}{d\beta_F} = \frac{1}{2(1-t)-\gamma(1-t+h)} \cdot \left[ 2(1-t) - \gamma(1-t+h) \right] > 0,
\]

\[
\frac{d^2\beta^D_j}{d(\beta_F)^2} = \frac{1}{2(1-t)-\gamma(1-t+h)} \cdot \left[ -16(1-t-h)^2 \right] \cdot \left[ 2(1-t) - \gamma(1-t+h) \right] > 0,
\]

\[
\left[ 4h^2 + 4 \left[ 2(1-t) - \gamma(1-t+h) \right] \cdot \left[ 2h + 2\beta_F(1-t-h) + \gamma(1-t-2h) \right] \right] > 0,
\]

\[
\left[ -3/2 \right. \left. \left[ 4h^2 + 4 \left[ 2(1-t) - \gamma(1-t+h) \right] \cdot \left[ 2h + 2\beta_F(1-t-h) + \gamma(1-t-2h) \right] \right] < 0,
\]

\( \beta^D_j \) is monotonically increasing and concave in \( \beta_F \). In sum, because \( \beta^F_j < \beta^D_j \) for \( \beta_F = 0 \), \( \beta^F_j \) is monotonically increasing and convex in \( \beta_F \), and \( \beta^D_j \) is monotonically increasing and concave in \( \beta_F \), a \( \beta_F > 0 \) exists for which \( \beta^F_j = \beta^D_j \). We label this threshold as \( \overline{\beta}_F \). Thus, for \( \beta_F < \overline{\beta}_F, \beta^F_j < \beta^D_j \) so that

\[
\beta^F_{D,1} < \beta^F_{D,2} < \beta^F_j < \beta^D_j < \beta^D_{D,1} < \beta^D_{D,2}
\]

and the MNC uses foreign ownership for \( \beta_D < \beta^D_j \), joint ownership for \( \beta_D \in (\beta^F_j, \beta^D_j) \), and domestic ownership for \( \beta_D \geq \beta^D_j \).

\[ \square \]

### 9.4 Proof of Proposition 2

Consider the equations (2), (3), (4), (6), (7), and (8). For \( \beta \leq \beta^F_j \), the MNC chooses foreign ownership and the MNC’s marketing activities are \( \theta^F_D(\gamma^F_j) + \theta^F_F(\gamma^F_j) \). The first derivatives of the MNC’s marketing activities with respect to \( \gamma \) and \( \overline{\gamma} \) are as follows:

\[
\frac{d}{d\gamma} \left( \theta^F_D(\gamma^F_j) + \theta^F_F(\gamma^F_j) \right) = \left\{ \begin{array}{ll}
0 & \text{for } \beta_D \leq \beta^D_{D,2} \\
\frac{\beta_F-1}{k} < 0 & \text{for } \beta_D \in (\beta^F_{D,2}, \beta^F_j]
\end{array} \right.
\]

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Consider the equations (2), (3), (4), (6), (7), and (8). For $\beta \leq \beta_F^D$, the MNC chooses foreign ownership. The first derivatives of domestic and foreign marketing activities with respect to $h$ are
as follows:

\[
\frac{d \theta_D^F (\chi^F)}{dh} = \begin{cases} 
0 & \text{for } \beta_D \leq \beta_D^E \\
\frac{- (1-t) \left[ 1 + \beta_D + \beta_F - (1 + 2 \beta_D) \beta_F^2 + \beta_F^4 \right]}{k \left( -1 - t - h - \beta_F^2 (1 - t - 2 h) \right)^2} < 0 & \text{for } \beta_D \in \left( \beta_D^{E,1}, \beta_D^{E,2} \right) \\
0 & \text{for } \beta_D \geq \beta_D^E,
\end{cases}
\]

\[
\frac{d \theta_F^E (\chi^F)}{dh} = \begin{cases} 
0 & \text{for } \beta_D \leq \beta_D^E \\
\frac{\beta_F (1 - t) \left[ 1 + \beta_D + \beta_F - (1 + 2 \beta_D) \beta_F^2 + \beta_F^4 \right]}{k \left( -1 - t - h - \beta_F^2 (1 - t - 2 h) \right)^2} > 0 & \text{for } \beta_D \in \left( \beta_D^{E,1}, \beta_D^{E,2} \right) \\
0 & \text{for } \beta_D \geq \beta_D^E.
\end{cases}
\]

For \( \beta \geq \beta_D^P \), the MNC chooses domestic ownership. The first derivatives of domestic and foreign marketing activities with respect to \( h \) are as follows:

\[
\frac{d \theta_D^P (\chi^P)}{dh} = \begin{cases} 
0 & \text{for } \beta_D \leq \beta_D^P \\
\frac{\beta_F (1 - t) \left[ 1 + \beta_D + \beta_F - (1 + 2 \beta_D) \beta_F^2 + \beta_F^4 \right]}{k \left( -1 - t - h - \beta_F^2 (1 - t - 2 h) \right)^2} > 0 & \text{for } \beta_D \in \left( \beta_D^{P,1}, \beta_D^{P,2} \right) \\
0 & \text{for } \beta_D \geq \beta_D^P.
\end{cases}
\]

\[
\frac{d \theta_F^P (\chi^P)}{dh} = \begin{cases} 
0 & \text{for } \beta_D \leq \beta_D^P \\
\frac{\left( 1 + \beta_F + \beta_F^4 + \beta_F (1 - 2 \beta_D) - 2 \beta_F^2 \beta_F \right) (1 - t)}{k \left( -1 - 2 t + \beta_F^2 (1 - t - h) \right)^2} > 0 & \text{for } \beta_D \in \left( \beta_D^{P,1}, \beta_D^{P,2} \right) \\
0 & \text{for } \beta_D \geq \beta_D^P.
\end{cases}
\]

\[
\square
\]

9.7 Proof of Proposition 5

Consider the thresholds \( \beta_D^F \) and \( \beta_D^P \) of equations (26) and (25). The first derivatives of these thresholds with respect to \( h \) are as follows:

\[
\frac{d \beta_D^F}{dh} = \frac{2 - \gamma + 2 \beta_F (1 - \beta_F + \beta_F \gamma)}{2(1 - t)} > 0,
\]

\[
\frac{d \beta_D^P}{dh} = \frac{1}{(2(1 - t) + \gamma(1 - t + h))^2} \left[ 2(1 - t) - \gamma(1 - t + h) \right] .
\]

\[
\left[ 1 + \frac{2(2 + h - 2 t) - 4 \beta_F \left( 1 - t - \gamma h \right) - \gamma \left( 6(1 - t) + 4 h - \gamma(1 - t + 4 h) \right)}{2 \sqrt{h^2 + \left( 2 \beta_F (1 - h) + 2 h + \gamma(1 - t - 2 h) \right) \left( 2(1 - t) - \gamma(1 - t + h) \right)}} \right] = 0.
\]
\[
+\gamma \left[ h + \sqrt{h^2 + \left(2\beta_F (1 - t - h) + 2h + \gamma(1 - t - 2h)\right) \left(2(1 - t) - \gamma(1 - t + h)\right)} \right] > 0.
\]

### 9.8 Proof of Findings in Section 7

First, we determine the marketing activities and royalty rates for all location possibilities. Second, we show that, by choosing foreign ownership, the MNC replicates the first-best solution when \( \beta_F = 0 \). Third, for two-sided spillovers, we provide an example in which the MNC optimally chooses headquarters ownership and an additional example in which the MNC optimally chooses domestic ownership.

#### 9.8.1 Domestic Ownership

The expected after-tax profits of the domestic and foreign division are as follows:

\[
E \left[ \Pi_{D,TSB}^D \right] = (1 - t - h) \left[ \theta_D + \theta_F \beta_F - \frac{k}{2} \theta_D^2 \right] + \gamma_i (\theta_F + \theta_D \beta_D) - (t + h) \gamma_r (\theta_F + \theta_D \beta_D),
\]

\[
E \left[ \Pi_{F,TSB}^D \right] = (1 - t) \left[ \theta_F + \theta_D \beta_D - \frac{k}{2} \theta_F^2 \right] - \gamma_i (\theta_F + \theta_D \beta_D) + t \gamma_r (\theta_F + \theta_D \beta_D).
\]

The divisions choose their marketing activities to maximize their own after-tax profits:

\[
FOC \theta_D : (1 - t - h) (1 - k \theta_D) + \gamma_i \beta_D - \beta_D \gamma_r (t + h) = 0
\]

\[
SOC \theta_D : (1 - t - h) (-k) < 0
\]

\[
FOC \theta_F : (1 - t) (1 - k \theta_F) - \gamma_i + t \gamma_r = 0
\]

\[
SOC \theta_F : (1 - t) (-k) < 0.
\]

Thus the FOCs determine a local maximum and the marketing activities are:

\[
\theta_{D,TSB}^D = \frac{1}{k} + \frac{1}{k(1 - t - h)} [\beta_D (\gamma_i - \gamma_r (t + h))],
\]

\[
\theta_{F,TSB}^D = \frac{1}{k} + \frac{1}{k(1 - t)} [t \gamma_r - \gamma_i].
\] (27)

The MNC’s expected overall after-tax profit is as follows:

\[
E \left[ \Pi_{HQ}^{D,TSB} \right] = (1 - t - h) \left[ \theta_{D,TSB}^D + \theta_{F,TSB}^D \beta_F - \frac{k}{2} \left( \theta_{D,TSB}^D \right)^2 \right]
\]

\[
+ (1 - t) \left[ \theta_{F,TSB}^D + \theta_{D,TSB}^D \beta_D - \frac{k}{2} \left( \theta_{F,TSB}^D \right)^2 \right] - h \gamma_r \left( \theta_{F,TSB}^D + \theta_{D,TSB}^D \beta_D \right).
\] (29)
Thus the FOC determines a local maximum and the optimal internal royalty rate is as follows:

$$\gamma = \frac{1}{1-t-h} \left[ -\beta_D^2 (1-t) \right] = 0$$

The headquarters chooses $$\gamma$$ to maximize the MNC’s expected overall after-tax profit:

$$E \left[ \Pi_D^{F,TSB} \right] = (1-t) \left[ \theta_D + \theta_F \beta_F - \frac{k}{2} \theta_D^2 \right] - \gamma (\theta_D + \theta_F \beta_F) + \gamma (t+h) \left[ \theta_D + \theta_F \beta_F \right].$$

The procedure of section 9.8.1 yields:

$$\theta_D^{F,TSB} = \frac{1}{k} + \frac{1}{k(1-t-h)} \left[ \gamma_r (t+h) - \gamma \right],$$

$$\theta_F^{F,TSB} = \frac{1}{k} + \frac{1}{k(1-t)} \left[ \beta_F (\gamma_t - t \gamma) \right].$$

The MNC’s expected overall after-tax profit is:

$$E \left[ \Pi_{HQ}^{F,TSB} \right] = (1-t) \left[ \theta_D^{F,TSB} + \theta_F^{F,TSB} \beta_F - \frac{k}{2} \left( \theta_D^{F,TSB} \right)^2 \right]$$

$$+ (1-t) \left[ \theta_F^{F,TSB} + \theta_D^{F,TSB} \beta_D - \frac{k}{2} \left( \theta_F^{F,TSB} \right)^2 \right] + h \gamma_r \left[ \theta_D^{F,TSB} + \theta_F^{F,TSB} \beta_F \right].$$

To maximize the profit shifted to the low-tax country, the headquarters sets the highest possible tax royalty rate, i.e., $$\gamma_t = \gamma$$. The procedure of section 9.8.1 yields:

$$\gamma_t = \frac{1}{1-t} \left[ (1-t) [\gamma_t - (1-t) \beta_D] + \beta_D^2 (1-t-h) \right]$$

and the MNC does not achieve first-best marketing activities.
9.8.3 Joint Ownership

The divisions’ expected after-tax profits are as follows:

\[ E \left[ \Pi_{D}^{J.TSB} \right] = (1 - t - h) \left[ \theta_D + \theta_F \beta_F - \frac{k}{2} \theta_F^2 \right], \]

\[ E \left[ \Pi_{F}^{J.TSB} \right] = (1 - t) \left[ \theta_F + \theta_D \beta_D - \frac{k}{2} \theta_D^2 \right]. \]

The procedure of section 9.8.1 yields that \( \theta_{D}^{J.TSB} = \theta_{F}^{J.TSB} = \frac{1}{k} \), which equals (18). The marketing activities are not equal to first-best marketing activities.

9.8.4 Headquarters Ownership

The divisions’ expected after-tax profits are as follows:

\[ E \left[ \Pi_{D}^{HQ.TSB} \right] = (1 - t - h) \left[ \theta_D + \theta_F \beta_F - \frac{k}{2} \theta_D^2 \right] - \gamma (\theta_D + \theta_F \beta_F) - \gamma (t + h) (\theta_D + \theta_F \beta_F), \]

\[ E \left[ \Pi_{F}^{HQ.TSB} \right] = (1 - t) \left[ \theta_F + \theta_D \beta_D - \frac{k}{2} \theta_F^2 \right] - \gamma (\theta_F + \theta_D \beta_D) + t \gamma (\theta_F + \theta_D \beta_D). \]

The procedure of section 9.8.1 yields that \( \theta_{D}^{HQ.TSB} \) equals (30) and \( \theta_{F}^{HQ.TSB} \) equals (28). The MNC’s expected overall after-tax profit is as follows:

\[ E \left[ \Pi_{HQ}^{HQ.TSB} \right] = (1 - t - h) \left[ \theta_D^{HQ.TSB} + \theta_F^{HQ.TSB} \beta_F - \frac{k}{2} \left( \theta_D^{HQ.TSB} \right)^2 \right] \\
+ (1 - t) \left[ \theta_F^{HQ.TSB} + \theta_D^{HQ.TSB} \beta_D - \frac{k}{2} \left( \theta_F^{HQ.TSB} \right)^2 \right] - h \gamma \left( \theta_F^{HQ.TSB} + \theta_D^{HQ.TSB} \beta_D \right). \] (33)

To minimize the profit shifted to the high-tax country, the headquarters sets the lowest possible tax royalty rate, i.e., \( \gamma = \gamma \). The procedure of section 9.8.1 yields:

\[ \gamma_{i}^{HQ} = \frac{1}{2 - 2t - h} \left[ \gamma (t + h)(2 - 2t - h) - \beta_F (1 - t - h)^2 - \beta_D (1 - t) \left( 1 - t - \gamma h \right) \right]. \] (34)

and the MNC does not achieve first-best marketing activities.

9.8.5 Foreign Ownership for \( \beta_{F} = 0 \)

For \( \beta_{F} = 0 \), the first-best marketing activities are \( \theta_{D.fb} = \frac{1}{k} + \frac{\beta_D(1-t)+h\gamma}{k(1-t-h)} \) and \( \theta_{F.fb} = \frac{1}{k} \), and \( \theta_{F.TSB}^{F} \) of equation (31) equals \( \frac{1}{k} \). With \( \gamma_{i}^{F} = \gamma - (1 - t) \beta_D, \ \theta_{D.TSB}^{F} \) of equation (30) equals \( \theta_{D.fb} \). Thus both divisions’ marketing activities under foreign ownership are first-best.
9.8.6 Headquarters Ownership

For $\gamma = 0.1$, $\overline{\gamma} = 0.3$, $t = 0.1$, $h = 0.15$, $k = 3$, $\beta_D = 0.8$, and $\beta_F = 0.6$, the MNC’s expected overall after-tax profits for each location possibility are as follows: $E \left[ \Pi_{HQ}^{D,TSB} \right] = 0.659$, $E \left[ \Pi_{HQ}^{F,TSB} \right] = 0.743$, $E \left[ \Pi_{HQ}^{I,TSB} \right] = 0.665$, and $E \left[ \Pi_{HQ}^{J,TSB} \right] = 0.796$. Thus the MNC optimally locates the intangible at the headquarters.

For $\gamma = 0.1$, $\overline{\gamma} = 0.3$, $t = 0.1$, $h = 0.15$, $k = 3$, $\beta_D = 0.1$, and $\beta_F = 0.9$, the MNC’s expected overall after-tax profits for each location possibility are as follows: $E \left[ \Pi_{HQ}^{D,TSB} \right] = 0.602$, $E \left[ \Pi_{HQ}^{F,TSB} \right] = 0.587$, $E \left[ \Pi_{HQ}^{I,TSB} \right] = 0.53$, and $E \left[ \Pi_{HQ}^{J,TSB} \right] = 0.572$. Thus the MNC optimally locates the intangible in the domestic division.

References


